Derivatives Summary

I. Basic understanding

- Financial Alchemy
  - **a)** CS + Call = no win
  - **b)** CS + Put = downside protection
  - **c)** Deposit PV of Exercise P + Call = downside protection

  - Put call parity – **Value of Call + PV of exercise P = Value of Put + stock P**
    - only works for European options with same exercise P
    - if Div → Value of Call + PV of exercise P = Value of Put + stock P – PV Dividends

- Remember
  - buying stock w/ call = buying on credit → Call Value is influenced by
    - interest
    - time (~ cumulative variability ~ σ t)
    - variance (b/c no downside to option itself))
  - option properties
    - upper bound = stock price
    - lower bound = payoff to exercise immediately (max of 0, stock P – exercise price)

- Variables in Option Pricing

Traditional Option	Observable?	Capital Project
i) C - Strike Price	Y	Amt expended (X)
ii) σ^2 - Risk of underlying asset	N	σ^2 of project returns
iii) r_F	Y	r_F
iv) $P_{S,0}$ – Current P of underlying asset	Y	Value of asset built (S)
v) t – time to maturity	Y	time can wait w/o losing opp

II. Valuing Options

- Replicating Portfolio
  - Option Value = **delta * (P_{today}) – PV of bank loan**
    - delta = hedge ratio = spread of possible option payoffs / spread of related possible share P's
    - bank loan = **(Δ $P_{\text{future pay off at time t}}$ - Option Payoff$_t$) / $(1+r)^t$**
      - = PV of difference b/w payoff exercising option (i.e., current P) and payoff from delta shs
    - option payoff = P - EX
  - How it works – find combo of stock & loan that replicates investment in option
    - Expected (Call Value) is same as % investment in stock and borrowing from a bank
      - i) if P falls → payback (bank loan + interest) w/ % of new stock P (~ bank loan negates value of % shs)
      - ii) if P rises → realize % value of new stock P – PV of bank loan (this = Call Value)
  - Why – O/W have arbitrage opportunity b/c can always sell Δ shs + buy call + lend the balance
  - Homemade option = buy or sell Δ shs and borrow / lend the balance

- Certainty Equivalent (Risk Neutral Valuation)
  - **Gultekin** – Call Value **= (1/δ) [$P_{sh\ now}$ – (P if depression / $1+r_F$)]**
    - δ = 1 / Δ = spread of related possible share P's / spread of possible option payoffs
    - r_F = P if depression / ($P_{sh\ now}$ – δ P_{Call})

  - **BM -** Option Value = **Expected future value / (1+r)**
    - Expected future value = [prob of rise * Option Payoff if sh P rises] + [(1-prob of rise)*Payoff if P falls]
    - probability of price rise (p) = [interest rate ($r^{1/m}$) – (- downside change %)] / [upside change – (- downside change)]
    - Option Payoff = P - EX
    - r_F = **(probability of P rise * % rise possible) + (1-probability of rise) * (- % decrease possible)**
      - = Expected return if assume investors indifferent to risk
      - % rise possible = total return / investment = (Value + dividend CF) / investment

- Binomial Method (p. 596) ~ discrete
  - based on isolating the probabilities of given up / down moves (similar to the methods discussed above)
  - more accurate than above methods given that sh P has infinite possible up / down movements

- Black-Scholes ~ continuous
  - Call value = [Δ * P] – [bank loan] **= [N(d_1) * P_0] – [N(d_2) * PV(EX)]**
    - Δ = N(d_1) = option delta
      - d_1 = **log [P_0 / PV(EX)] / [σ $(t)^{0.5}$] + [σ $(t)^{0.5}$]/ 2**
        - EX = exercise P of option
        - PV(EX) = EX / $(1+r_F)^t$ = Ce^{-rt}
        - t = periods to exercise date
        - σ = σ per period of continuously compounded rate of return on stock
        - P = current stock P
        - log ~ natural log
      - N(d) = cumulative normal probability density function (~ prob that normally distributed RV will be ≤ d)
    - bank loan = **N(d_2) * PV(EX)**
      - d_2 = d_1 - [σ $(t)^{0.5}$]
      -
  - what it does – replicates call option by levering investment in stock continuously (not at intervals)
  - assumes[1] –
    - The prices of the underlying asset follow an Ito process. (See Hull, 196 - 198)
    - The option can be exercised only on its expiration date (European option).
    - Short selling is permitted.
    - There are no transaction costs.
    - All securities are divisible and pay no dividends.
    - There is no riskless arbitrage.
    - Trading is a continuous process.
    - The risk-free interest rate is constant and remains the same for all maturities.
  - remember
    - lognormal distribution – better approximates stock changes than normal distribution (P never goes down by more than 100% but small chance can go up by much more than 100%)

- Relative values
  - American Call / no div = European Call (b/c should not be exercised before maturity if no dividends – or would more accelerate reduce the value of the call, as it increases the longer t to maturity)
  - European Put / no div = Value of Call – Value of Stock + PV (Exercise P)

  - American put / no div = always more valuable than European Put (b/c can re-invest the proceeds at an earlier time)
    - not valued by Black Scholes b/c it does not allow for early exercise

  - European Call with div = Black Scholes value – PV(Div) (~ b/c call holder does not get div)

  - American Call with div – always worth more alive than dead (keep option open + earn int on exercise $)

[1] http://www.mathworks.com/access/helpdesk/help/toolbox/finance/finance.shtml

- only valued with step by step binomial method (constantly check to see if more valuable if exercised just before the ex-dividend date

- remember
  - key pt (for any option valuation) = portfolio of underlying stock and investment that replicates payoff from option
  - DCF <> work b/c there is no correct discount rate
    - option is always riskier → higher Beta & σ
    - option risk changes every time P changes
  - consider interest → delay exercising call (interest free loan) → accelerate exercising put (to re-invest)

III. Real Options

- key parameters
  - P = value of underlying asset (~ thru DCF)
  - σ ~ from comparables ~ traded stocks w/ business risks similar to the opportunity (s/b unlevered to eliminate risk re financing)
  - r_F – discount future exercise P at risk free rate (if uncertain → you are really trading one risky asset for another)

- Capital Projects
  - DCF – applies if project cannot be delayed (t = 0) or has no variance (σ = 0)
    - NPV_q = PV (CF) / PV (Capex)
    - RoT ~ invest if $NPV_q > 1$

  - option – when project c/b delayed
    - **option value** = look up % X underlying Asset Value (S) for NPVq and cumulative variance ($\sigma (t^{0.5})$)
      - cumulative variance = σ^2 X t (~ variance of project times time remaining on project)
        - measures how much things could change before time runs out
    - NPV_q – still matters, but is influenced by riskiness of project
    - RoT - capital project s/b pursued if PV(CF) > PV (Capex) at maturity of option

- remember
  - RoT → accept if APV is positive (APV = NPV + call Value)
  - if have to commit to project now → consider if option is out of money or not (what is NPV)
  - Timing option - CF ~ dividend on an American call (if no CF, hold to mat.; if CF, may be exercise option before mat.)
    - key = ability to wait and see
    - ~ Call that is in the money
  - Abandonment option ~ Put (binomial method)
  - option value = EV (project w/ option) – EV (project w/o option)
  - with B-S → always overstate value of real option

IV. Warrants & Convertibles (Chrysler)

- Warrants
  - sh P_{after} conversion = [Value of Equity $_{old}$ + (Number Warrants * Exercise P)] / (Old number of shs + number warrants)
  - Warrant Value$_{maturity}$ (W) = max (P_t – EX, 0) = [1 / (1+# warrants per sh)] * max [0, ((Value of Equity / Old # shs) – EX))]
    - = 1 / (1+q) * max (V/N – EX, 0)   (~ where V/N = per share value of similar firm w no O/S warrants)
    - = value of [1 / (1+# warrants per sh)] call options written on stock of alt firm w/ no O/S warrants
    - <u>iterative approach</u>
      - i) find / assume n, M, T, σ, rF   [~ NOTE – σ = σ (E+N) BUT maybe use σ (E) as proxy]
      - ii) assume a $W_{assumed}$
      - iii) $P_{stock\ now}$ = P $P_{stock\ now}$ + (M/N) $W_{assumed}$
      - iv) Value P_{Call} (using assumptions from i and ii above)
      - v) Evaluate W_{actual} = N / (N+M) * P_{Call} → does it equal $W_{assumed}$
        - Yes – you are done
        - No – go back to (ii) and adjust the $W_{assumed}$

  - dilution factor **= M / (M+N)**
    - N = O/S shs
    - M = O/S warrants allowing purchase of one sh per warrant
    - W = value of Warrant

- Remember –
  - value warrant with B&S by adjusting for dilution

- Convertible Bonds –
  - conversion ratio = # shs into which each bond c/b converted