# Funding Investments – Overview (Concise)


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**Key questions**


*How value C(T)??* ........ *10*
*Why partial hedge??* ........ *12*


## General Issues

- **Conventions** -
  - Actual / Actual ~ US treasury
  - Actual / 365 ~ Eurobonds, Euro floating rate notes ,foreign gov't bonds
  - Actual / 360 ~ LIBOR, Eurodollar deposits, commercial paper, banker acceptance, repos,
  - 30/360 ~ corporate bonds, US agency bonds, muni bonds, mortgages

  - conversion issues
    - money market basis ~ ACT / 360
    - bond basis ~ ACT / 365 ~ ACT / ACT ~ 30 / 360 (all being equivalent in a non-leap year)

    - process – Always do in the following order …
      - **i)** convert available info to desired period for the given basis →
        - semi-annual to annual → $y^* = (1+y/2)^2 - 1$
        - annual to semi-annual → $y^* = [(1+y)^{0.5}-1] * 2$
      - **ii)** convert (i) to desired bond / MM basis →
        - bond to MM basis → **M = y* 360 / 365**
        - MM to bond basis → **B = y* 365 / 360**

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- **Discount Rates** –
  - spot rate of interest ($r_n$) = $\dfrac{1}{(1+y/k)^{kn}} = \left[\dfrac{1}{z^{k/n}}\right] - 1$ (k ~ compounding periods / yr) (n ~ compounding periods in term)

    - = yield on a zero → w/ equation $P_n = 100 / (1+y_n/2)^n = 100 * z_n$ → $y_n = 2 * [(100P_n)^{1/n}] - 1$

    - derived by → computing a zero from coupon bond →
      - **i)** find discount factor (z) included in Price with (final) payment **+**
      - **ii)** work backwards, finding discount factor implied by intermediate payments **+**
      - **iii)** convert discount factor to yield

- discount factor (z) = $\frac{1}{(1+r)^{n/k}}$ for a zero bond **= P / 100 = PV ($CF_n$) / $CF_n$**
  - derived by – bootstrapping from swap rates
    - FIRST → **z(0,0.5) = 1 / (x * 0.5)**
    - THEN → **$z(0,T_i) = \frac{1 - x(T_{j+1}) \, sum^{j}_{i=1} [z(t,T_i)]}{1 + x(T_{j+1})}$**
  - compounding periods -
    - semi-annual compounding → $z_n = P_n / 100 = 1 / (1 + y_n / 2)^n$
    - annual compounding → $z_n = 1 / (1 + y_n)^n$
    - "k" compounding → $z_n = 1 / (1 + y_n / k)k^n$
    - continuous compounding → $z_n = e^{-ny}$
    - "simple interest" → $z_n = 1 / (1 + ny_n)$
    - "discount basis" → $z_n = (1 - ny_n)$
- forward rate- $f_t(T_1,T_2) = \left[\frac{(1+r_2)^{T2-t}}{(1+r_1)^{T1-t}}\right]^{[1/(T2-T1)]} - 1 = [z(t,T_1) / z(t,T_2)] - 1$ (~ consider need to annualize)
  - rate at which 2 parties agree to lend / borrow money for specified period of time in the future
  - explicit - if embodied in K
  - implicit – as reflected in the term structure of interest rates
- Yield ~ just way to quote Bond's P (NOT relative measure of value)
  - *current yield* = coupon rate / (Clean P / 100) ~ ignores TVOM, focusing only on coupons received
  - *par yield* ~ YTM of coupon bearing bond priced at par

## Straight Corporate Bonds

- **Zero Coupon Bonds**
  - gross (unannualized) rate of return **= F / P – 1** (where F = future payoff and P = initial investment)
- **Coupon bonds =**
  - Pricing -
    - general bond equation ~ **$P + AI = [(C/k) / (1+r/k)^{1/k}] + [(C/k) / (1+r/k)^{2/k}] + \ldots + [(C/k + F) / (1+r_m/k)^m]$** (C ~ coupon payment;
      - BUT assumes no default
    - using zeroes –
      - $P_n$ = **periodic payments (C) [z(t,1) + z(t,2) + … z(t,n)] + final payment (F)z(t,n)**
        - think of coupon bond as a series of zeroes
      - $P_t$ = **$sum_{j=1}^{n}$ (C/2) z(t,j) + Fz(t,n)**
    - *Clean price* (P) ~ quoted price
    - *Dirty price* (P + AI) ~ clean price + accrued interest
    - Accrued interest ~ **["Days" elapsed / "Days" in period] * coupon interest**
  - Duration ~ **sum of [Time to each CF * (PV of each future CF / P)] = Σ of [Time to CF * ((C/ $(1+y)^t$) / P)]**
    - ~ measured in yrs
    - weighted average of times in future when CF are t/b received (~ measured in yrs)
    - *immunization* (against changes in interest rates) ~ matching bond duration w/ needed CF
    - McCaulay Duration **= $[(1+y) / y] - [1+y+n(C/F-y)] / [C/F((1+y)^n - 1)+y]$** (C = Ann. Coupon Amt; F =Face)
      - ~ % change in Bond P relative to % change in (1+YTM)
    - Duration of perpetuity = **(1+y) / y**
    - Duration of a portfolio = **$w_X D_X + w_Y D_Y$** = $[(\text{sum of } (CF_X + CF_Y)) / (1+y)^t] / (X+Y)$

- Duration for Hedging – $H = (P_{LTBond} / P_{STBond}) * (D_{LTBond} / D_{STBond}) * [(1+ Y_{STBond}) / (1+ Y_{LTBond})]$

- remember - equals maturity only with zero coupon bonds
  - duration of zero coupon bond = its maturity
  - w/ constant maturity → lower coupon rate means a higher duration (?)
  - w/ constant coupon rate → longer maturity means higher duration (always for bonds selling at par or prem)
  - w/ all else constant → lower yield means higher duration
  - as term structure of int rates falls, durations lengthen

## Government & Foreign Bonds

- **Treasury Bills – P = 100 – (D/360) * R** [D ~ days to maturity; R ~ % discount]
  - *consist of* - non coupon securities + 12, 26, 52 week maturity + auctioned every 4 weeks + bought / sold on bond discount yield basis (~ 360 day yr)
  - true bond equivalent yield (Y) ~ **(annualized interest *100) / P**
    - < 6 mos to maturity → Y = **[(365 / D) * (100-P) * 100] / P**
    - > 6 mos to maturity → Y = $[(-2D/365)+ 2*[(D/365)^2 – ((2D/365)-1) * (1-1/P)]^{0.5}] / [(2D / 365)-1]$ ??
    - if have discount rate (R) → Y = **(365 * R) / [360 – (R * D) / 100]**
    - always higher than discount yield

- **Treasury Notes** –
  - < 6 mos ~ **P = (1+C/2) / [1+Y(D/2B)]** ??
    - B = days in the current coupon period; N = number of remaining coupons)
  - > 6 mos ~ $P = (C/2) / (1+y/2)^{D/B} + (C/2) / (1+y/2)^{1+(D/B)} + … + (C/2) / (1+y/2)^{N-2+(D/B)} + (100 + C/2) / (1+y/2)^{N-1+(D/B)}$
  - *quote* ~ flat P ~ clean P (P') **= P- C/2 * (B-D)/B**
  - *consist of* - coupon securities + 2-10 yr maturity + int paid semi-annually

- **Treasury Bonds** –
  - *consist of* – coupon securities + > 10 yr maturity + same P and Y conventions as notes
  - if issued pre-1984 → are callable at par (plus AI) in last 5 yrs to maturity
  - flower bonds – can be applied at par + AI to a dededents estate taxes (trade at lower coupon and Y)

- **STRIP** – coupon and principal of >10 yr Bond that is traded separately (since Oct. 1984)
  - $P = 100 / (1+Y/2)^{N-1+(D/B)}$ (N~ number of 6 months periods in remaining life of STRIP)

- **Eurobonds** –
  - int. accrues annually

## Swaps

- **Swaps** -
  - Swap rate (X) payable at each date → equates fixed and floating CF →computed through any of following …

    - <u>method 1</u> - fixed rate paid on same dates as floating rate such that PV(floating rate CF) = PV(fix rate CF)
      - where $X = \frac{\text{sum of}^N_{i=1} [\text{forward rate}_{ki} * z(0,k_i)]}{\text{sum of}^N_{i=1} (z_0,k_i)}$ (~PV floating rate CF) / (~PV fixed rate CF)

    - <u>method 2</u> – replicate floating rate CF (since we know fixed rate CF)
      - conceptually – PV (invest $1 + receive interest on $1 + re-invest $1 at reset rate + … + receive interest & principal + assume pay off previously issued zero to eliminate $1 principal)
      - such that – **X for maturity T =** $\frac{[1 – z(0,T)]}{\text{sum}^T_t [z(0,t)]}$ (~PV floating rate CF) / (~PV fixed rate CF)

      - <u>key point</u> – swap rate (X) = T year par bond yield

    - <u>method 3</u> ~ **interest rate that causes the fixed rate bond to sell at par** (given the yield curve implicit in the PV)

- asset swaps – pkg allowing investor to buy fixed rate bond + hedge almost all int rate risk by swapping fixed to floating payments (but retains CR risk) → essentially ~ when investor (rather than borrower) utilizes a swap
    - replication – asset swap buyer pays par up front + receives fixed rate bond from asset swap seller + enters int rate swap to pay seller fixed coupon equal to asset (w/ asset swap buyer receiving floating rate payments + / - agreed spread)
    - break-even swap spread ~ swap spread that makes PV of all CF = 0
- day convention issues -
    - money market basis ~ ACT / 360 (assuming 30 day month)
    - bond basis ~ ACT / 365 ~ ACT / ACT ~ 30 / 360 [all being equivalent in a non-leap year]
    - process – Always do in the following order …
        - **i)** convert available info to desired period for the given basis →
            - semi-annual to annual → $\mathbf{y^* = (1+y/2)^2 - 1}$
            - annual to semi-annual → $\mathbf{y^* = [(1+y)^{0.5}-1] * 2}$
        - **ii)** convert (i) to desired bond / MM basis →
            - bond to MM basis → $\mathbf{M = y^* 360 / 365}$
            - MM to bond basis → $\mathbf{B = y^* 365 / 360}$

## Options

- **Put-call parity** ~ $\mathbf{V_{call} + Ke^{-r(T-t)} = V_{put} + V_{sh\ of\ stock}}$
    - $C \geqslant 0$ – value of call is never less than 0
    - $S \geqslant C$ – value of call never more than stock itself
    - $C \geqslant S - Ke^{-r(T-t)}$ – value of call never less than stock P minus PV of exercise P
    - $C(K_1) \geqslant C(K_2)$ if $K_2 > K_1$ - value of call never less than identical call w/ higher strike P
    - $K_2 - K_1 \geqslant C(K_1) - C(K_2)$ if $K_2 > K_1$ - difference in value of identical calls never > diff in strike Ps
    - $C(t_2) \geqslant C(t_1)$ if $t_2 > t_1$ - value of call never less thab value of identical call w/ shorter t to expir.
- **Black Scholes** ~
    - $\mathbf{P_{Call} = S * N(d_1) - (Ke^{-rt}) N(d_2)}$
        - $d_1 = [\ln(S / K) + (r_F + (\sigma^2/2))\, t] / \sigma(t^{0.5})$ (note – assumes constant $\sigma^2$, but may not be)
        - $d_2 = d_1 - \sigma(t^{0.5})$
        - $N(d_1)$, $N(d_2)$ – value of cumulative normal distribution at $d_1$ and $d_2$
    - $\mathbf{P_{Put}} = \mathbf{V_{call} + Ke^{-r(T-t)} - V_{sh\ of\ stock}}$
- **Binomial Option Pricing Model**
    - Pricing Option (against stock share) ~ $\mathbf{P_{Call} = \Delta S - B}$ (S ~ P of sh of stock)
        - $\Delta = [C_U - C_D] / [S_U - S_D]$ (number of shares of stock bought to replicate the call)
        - $B = [C_U S_D - C_D S_U]$ / $\mathbf{[}(S_U - S_D)(1+r)\mathbf{]}$ (B ~ amount borrowed – necessary to replicate exercise P)
        - $C_U = \Delta S_U - B(1+r)$
        - $C_D = \Delta S_D - B(1+r)$
    - Pricing Option (against Value of firm w/ debt) ~ $\mathbf{E = \Delta V_{firm} - B}$ (E ~ Value of equity)
        - $\Delta = [E_U - E_D] / [V_U - V_D]$ (number of shares of stock bought to replicate the call)
        - $B = [E_U V_D - E_D V_U]$ / $\mathbf{[}(V_U - V_D)(1+r)\mathbf{]}$ (B ~ amount borrowed – necessary to replicate exercise P)
        - $E_U \sim \max(V_U - F, 0) = \mathbf{\Delta V_U - (1+r)B}$
        - $E_D \sim \max(V_D - F, 0) = \mathbf{\Delta V_D - (1+r)B}$
    - Pricing Debt ~ $D = \mathbf{N(E+D) - B = NV_{firm} - B = [N / (1- N)]\, V_{firm} - [1 / (1- N)]\, B}$
        - $N = \mathbf{[D_U - D_D] / [V_U - V_D]}$ (N ~ Δ)
        - $B = \mathbf{[D_U V_D - D_D V_U] / [(V_U - V_D) * (1+r)]}$ (B = amt borrowed)
        - $D_U = \min(V_U, F, 0)$
        - $D_D = \min(V_D, F, 0)$
        - $V_{firm} = E + D$

- essentially says → risky debt CF ~ replicated by <u>buying</u> [N / (1-N)] $V_{firm}$ and <u>lending</u> [1 / (1- N)] * B

- <u>Inputs</u> –
  - **i)** stock returns = log normally distributed (skewed right and never less than -100%) – $r_t = \ln (P_{t+1} / P_t)$
  - **ii)** volatility →
    - **1)** daily log price relative ($r_t$) = $\ln (P_{t+1} / P_t)$
    - **2)** est. annual variance of $r_t \sim \sigma^2_{annual} = (250)^{0.5}\sigma^2_{daily}$ [250 ~ typical trading days / yr]
  - **iii)** up factor = $\exp [\sigma(T/n)^{0.5}]$ (~ n = number of intervals that T is split into = BS if n ~ infinity)
  - **iv)** down factor = 1 / (up-factor)

- **Delta Hedging** (with options)
  - hedge ratio (h) ~ option Δ = Δ Option P / Δ Underlying Asset P
    - Δ of long position **=** $\mathbf{h^{-1}}$ (~ amount of options to sell short)
    - Δ of short position = - $h^{-1}$
    - Δ of call option = N(d1)
    - Δ of put option = N(d1) - 1
  - delta neutrality – requires that – **100 Δ + N = 0** (where N = # shs bought; 100 used b/c 100 options in typical K)
  - self financing – requires that 100N + M = BS option P * 100 (where M ~ amt borrowed / invested $r_f$)

- **Equity as an option** –
  - K = face value of debt
  - value of stock ($S_t$) = $\mathbf{V_t\,N(d_1) - F\,N(d_2)\,e^{-rft}}$ (where V ~ value of firm assets; F ~ Debt Face)
    - $N(d_1) = \mathbf{[\ln(V/F) + (r_f + 0.5\sigma^2)\,t\,] / \sigma\,(t)^{0.5}}$
    - $N(d_2) = \mathbf{d_1 - \sigma\,(t)^{0.5}}$
  - $S_t$ increases as ~ F increases; $T_{debt}$ – t increases; $\sigma^2$ of V increases; $r_f$ increases; value of firm assets increases

- **Debt Valuation** –
  - Value of Call = Value of (Put + Underlying Asset) – PV of exercise P
  - Value of Underlying Asset = Value of Call + **[**PV (F) – Value of Put**]** [] = Value of Debt
  - Value of Firm's Assets = Value of Equity + **[**PV (F) – Put Value**]** [] = Value of Risky Debt
  - Value of Risky Debt = Value of Default free bond – Value of Put
  - Value of Debt ~ $\mathbf{V_{firm,\,t}}$ **– Equity ~ $\mathbf{V_t - C(V_t;F,\,\sigma,\,t)}$**
    - at payoff – **D* = Min[F, V*]**
    - prior to maturity – $\mathbf{D_t}$ **~ PV(F) – Value of put = $\mathbf{F\,e^{-rt} - P(V,F,t)}$**

- **Junior Debt Value**– $D_J$ **= $\mathbf{Max[Min(V^*\text{-}F_S,F_J),0] = C(V;\,F_S) - C(V,\,F_S + F_J)}$**
  - $V = D_S + D_J + S$
  - $D_S = V - C(V;F_S)$
  - $S = C(V;\,F_S + F_J)$
  - $V = [V - C(V;F_S)] + D_J + C(V;\,F_S + F_J)$

- **Risky Debt** ~ Value of Default free bond – Value of Put ~ $\mathbf{D_r = F\,e^{-rt}}$
  - value of default free bond ($D_f$) **= $\mathbf{F\,e^{-rft}}$**
  - yield on risky debt (r) ~
    - **r = ln [F/D]/t**
    - $\mathbf{r = r_f - \{\ln[L^{-1}\,N(d_1) + N(d_2)]\} / t}$ (where $L = D_f / V$) *~ Garbade*
  - credit spread – difference b/w promised yield on risky and risk free debt ~ $\mathbf{r - r_f}$

- **Risk Free Debt** ~ $\mathbf{D_f = F\,e^{-rft}}$

## Embedded Options

- **Callable Bonds** -
    - Proof of relationships → identify transactions allowing arbitrageur to realize positive CF today w/o Δ in future CF
        - **if $P_C < P_{NC} - C$ →**
            - purchase callable bond for $P_C$
            - purchase option on bond for C (~ allows to cover short position)
            - sell short non-callable bond for $P_{NC}$ (~ for more than $P_C$ + C)
        - **if $P_C > P_{NC} - C$ →**
            - purchase non-callable bond for $P_{NC}$
            - purchase option on bond for C (~ allows me to call if company calls bond form me)
            - sell short callable bond for $P_C$ (~ for more than $P_{NC}$ + C)
        - key point → arrange such that revenue on initial transaction is a riskless arbitrage
            - but option does not trade → so would have to buy / sell portfolios of other securities that replicate the option
    - Pricing -
        - Black Scholes – does not work b/c no closed form solution if can be called at any time for varying strike P (K*)
        - Binomial Option Pricing Model –
            - Value of D -
                - at interim ~ **Max [λ V, Min(call P, D+C)]**
                - at maturity ~ **Max [λ V, Min (V, D+C)]**
            - assume Bond called if value + accrued interest > call P
- **Convertible Bonds** –
    - Pricing –
        - Black Scholes –
            - conversion ratio (r) ~ # shs of C/S obtained upon surrender of the convertible debenture
            - conversion P = **Face Value / conversion ration = K / r** (K = face value)
            - new shs issued on 100% conversion = **Mr** (M = # convertibles outstanding)
            - dilution factor (λ) ~ equity % owned by converting B/H = **Mr / (N + Mr)** (N = # shs O/S pre-conversion)
                - anti-dilution clauses – typically protect conv. bonds from dilution if shs issued at below market
            - conversion RoT – convert only if ~ **λV* > K**
        - Binomial Option Pricing Model –
            - w/ call and convertability → find value of convertability by solving for D vs. straight Debt
            - Value of D
                - at interim ~ **Max [λ V, Min(call P, D+C)]**
                - at maturity ~ **Max [λ V, Min (V, D+C)]**
- **Warrants** –
- **PERCs** –
    - Pricing – **PK = PV (div on PERC) – PV (div foregone on C/S) + [P – Call]** [P-Call ~ reflects ltd upside]

## Credit Risk: Bond Ratings and Spreads

- credit risk – likelihood that obligor will not honor a promised debt payment (chance of default by an issuer)
- relative risk – refers to ordering of risks
- absolute risk – refers to ability of rating t/b guide to E(loss)
- junk bond – bond w/ at least one rating below BBB- level from either S&P or Moody's

- rating drift = ***(# upgrades - # downgrades) / # rated issues*** (~ summarizes overall incr. / decr. in CR quality as % of one letter grade)

- rating activity = ***(# upgrades + # downgrades) / # rated issues*** (~ shows fraction of rated issues ~ re-rated)

- bond spread ~ (YTM for a corp bond) – (YTM of equiv. maturity (or duration) Treasury or LIBOR rate bond)
  - spread ~ compensation for risk of default ~ viewed as indicator of default (increase means > possibility of default)
  - actual value as indicator → aggregate factors across firms are more important than firm specific factors (p. 399)

- **Why do firms pay for ratings**? – release inside info + rating agency doing service to investors/firm helps investors invest

- **causality – is there info in ratings**? – empirical studies on yield spreads show causality from ratings to Ps of bonds
  - study of effect of bond downgrades shows negative correlation w/ bond Ps (but is it the downgrade or the related negative event?)
  - study of effect of Moody's providing more info in ratings → information in ratings changes ~ re default risk (and largely diversifiable)

## Pricing Risky Bonds – using bond ratings

- **probability method**
  - variables -
    - c ~ coupon rate
    - $\delta$ ~ recovery rate ~ % recovery of promised int + principal promised on a bond that defaults
    - p ~ credit risk ~ probability of default
    - z ~ discount factor ~ [1 / (1+r)]
    - $\lambda$ ~ loss rate ~ 1-$\delta$
    - q ~ probability of an uptick in value for BOPM purposes

  - expected future loss = $100p(1+c)(1-\delta)$

  - expected rate of return $(1+r) = \{(1-p)(1+c) + p[\delta(1+c)]\}$ ← ← ← ← ← ←← ←← ←← ←← ←← ←← ←

  - pricing equation –
    - → $1+r = (1+c)\,[p\delta + 1-p] = (1+c)[1-p(1-\delta)]$ → $1+r = (1+c)(1-p\lambda)$
    - → $c = [(1+r)/(1-p\lambda)] - 1$ → $c = (r+p\lambda) / (1 - p\lambda)$
    - → $p = (c-r) / \lambda(1+c)$ → $p = (c-r) / \lambda(1+c)$

  - credit spread $(c-r) = p\lambda(1+r) / (1- p\lambda)$

- **BNOP** –
  - $Value_{call}$ (C) = $[pC_U + (1-p)\,C_D] / (1+r)$ → says C = $(Prob_{uptick} * V_{call\ on\ uptick} + Prob_{downtick} * V_{call\ downtick}) / (1+r)$
    - p ~ true probability of security P moving up / down IFF people are really risk neutral
    - ***key point*** – said 3 different ways …
      - BNOPM solution for value of call = value of call in risk neutral economy
      - attitudes re risk <> matter for option P
      - ***valuation based on no arbitrage b/w replicating portfolio and the security is identical to risk neutral valuation***

  - causality – derives from returns on underlying asset at each binomial outcome to probability (p) – not vice versa
    - → risk neutral probabilities (p) ~ probabilities that make expected return of an asset = $r_f$
    - → p must solve $(p)u + (1-p)\,d = 1+r$ → $p = \dfrac{(1+r-d)}{(u-d)}$
      - u ~ (1 + RoR on underlying asset in up node) – w/ RoR calced over period that up / down move can occur

- d ~ (1 + RoR on underlying asset in down node)

- this risk neutral valuation – c/b applied wherever (i) replicating portfolio c/b constructed OR (ii) risk neutral (p) c/b inferred from traded securities OR (iii) investors risk ~ neutral re their risk prefs

- **Transition Probabilities & Matrices** ~ using Markov Processes

  - recovery rates (δ) ~ amount B/H receives at default (determined mainly by – bond seniority + rating at default)
    - varies over time (due to effect of business cycle, change in bankr. laws, etc.)

| Debt Senior. | Recovery Rate for Given Year of Maturity* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 74 - 91 | 1984 | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 |
| Sr. Secured | 59.02 | 76.04 | 64.30 | 87.78 | 76.29 | 74.63 | 33.72 | 60.81 | 67.13 |
| Sr. Unsecured | 46.56 | 69.97 | 33.65 | 67.39 | 44.57 | 43.26 | 35.29 | 45.50 | 46.53 |
| Sr. Subord. | 45.69 | 32.87 | 28.84 | 47.73 | 39.85 | 40.27 | 26.47 | 33.68 | 32.10 |
| Subordinated | 39.32 | 33.93 | 31.14 | 37.75 | 35.03 | 25.95 | 23.39 | 16.58 | 26.44 |
| Jr. Subord. | - | - | - | - | 55.00 | 17.50 | 18.86 | 3.63 | 17.95 |
| Wgted Avg. | 48.83 | 40.29 | 35.76 | 72.57 | 44.29 | 39.11 | 28.17 | 32.65 | 39.22 |

* of corp defaults in given yr, this is avg P of bonds that traded w/i 2 weeks of announced default (~ market expectation)

  - transitional probabilities – prob that bond of particular rating will move to another rating over some indicated time

  - Markov processes – evolution of transition probabilities

    - $P_{risky\ bond}$ = $\boldsymbol{P_{riskless\ bond}}$ *** * [Par – (chance will default(Q) X expected loss)]***
      - $D^j_{(t,T)} = D^f(t,T) * [1 - Q_{(t<T)}(1-\delta)]$ ← ← ← ← ← ←← ←← ←← ←← ←← ←← ←

    - remember
      - once a bond enters default it stays there → any emergence from bankruptcy is effectively new bonds
      - $Q_{i,j}$ ~ transition probability matrix (the area w/ the probabilities) – i = row; j = column
      - $Q^x$ ~ matrix Q multiplied by itself (x) times

    - avg one year transition prob (1981 – 1991)

| Initial Rating | Rating at End of Year | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | AAA | AA | A | BBB | BB | B | CCC | D | NR |
| AAA | 0.875 | 0.095 | 0.008 | 0.002 | 0.003 | - | - | - | 0.018 |
| AA | 0.008 | 0.879 | 0.073 | 0.010 | 0.003 | 0.003 | - | - | 0.025 |
| A | 0.001 | 0.028 | 0.861 | 0.063 | 0.010 | 0.004 | - | 0.001 | 0.032 |
| BBB | 0.001 | 0.004 | 0.062 | 0.800 | 0.061 | 0.015 | 0.002 | 0.004 | 0.055 |
| BB | - | 0.002 | 0.007 | 0.065 | 0.720 | 0.094 | 0.012 | 0.022 | 0.097 |
| B | - | 0.002 | 0.003 | 0.006 | 0.045 | 0.72 | 0.038 | 0.060 | 0.127 |
| CCC | - | - | 0.010 | 0.010 | 0.018 | 0.067 | 0.573 | 0.204 | 0.118 |

## Credit Default Swaps (as a Credit Derivative)

- credit derivative ~ security w/ payoff contingent on a [credit rating (rare), default performance (more), or return performance] of underlying reference asset
- credit default swap ~ agreement where one party purchases insurance against default of specified reference credit / security / group of securities
- protection seller ~ long in credit exposure (gets premium for potential payment to protection buyer if credit event occurs)
- premium ~ swap quote ~ ***paid semi-annually / quarterly in arrears on actual / 360 basis***
  - ~ expressed in bps per annum over swap tenor

- **uses** - hedging an exposure + creating synthetic floating rate notes (assets) + creating synthetic short position

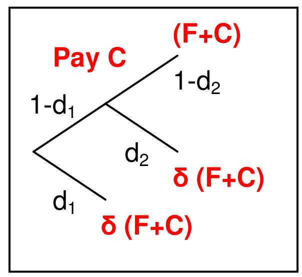
- **cash flows** -
  - <u>INCEPTION</u> - no exchange of principal

  - <u>PERIODIC payment to protection seller</u> ***= notional amt * bid-ask amt (in bps) * year convention adjustment***

  - <u>DEFAULT payout to protection buyer</u> ~ depends on the type of instrument
    - cash settled payout → ***notional amount * max [(100-$P_{default}$),0] / 100***
    - binary payout → ***notional amount * max [(100-x),0] / 100*** (x ~ set in advance, ignoring events)
    - physically settled → ***protection buyer has right to put defaulted security to protection seller at par***
      - more common than cash settled (applies to all single name credit default swaps)
      - risk = short squeeze

- **Swap Pricing – by replication** – DOES NOT WORK (too many assumptions needed – no LT short in corp bonds + often no trade in underlying asset)

- **Bond Pricing – Model based** – $P = \sum^n_{t=1} \frac{s_t C + s_{t-1} d_t \delta (F+C)}{(1+r)^t} + \frac{s_n F}{(1+r)^n}$




$$P = s_1 C z_1 + s_0 d_1 \delta(F+C) z_1 + s_2 (F+C) z_2 + s_1 d_2 \delta (F+C) z_2$$

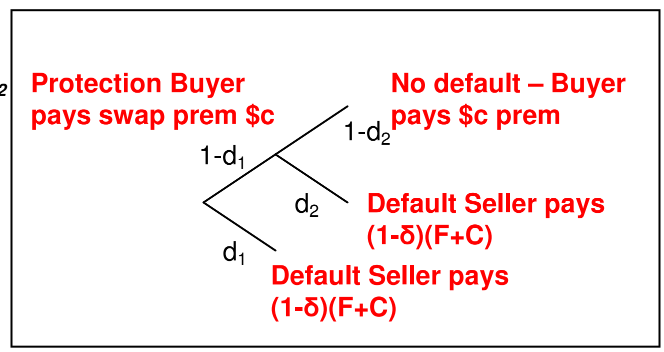
  - $p_t$ ~ cumulative prob of defaulting up to just prior to date t
  - $d_t$ ~ prob of defaulting in next period given survival to date t
  - $s_t$ ~ prob of surviving (not defaulting) to date t ~ $1 - p_t$ ~ $(1-d_1)(1-d_n)$ [~ EXCEPT → $s_1 = (1-d_1)$]
  - requires – risk neutrality

- **Swap Pricing – Model based** –

    - $c(s_1 z_1 + s_2 z_2) = s_0 d_1 (1-\delta)(F+C) z_1 + s_1 d_2 (1-\delta)(F+C) z_2$
      (to seller) (to buyer)

    - $c = \frac{s_0 d_1 (1-\delta)(F+C) z_1 + s_1 d_2 (1-\delta)(F+C) z_2}{(s_1 z_1 + s_2 z_2)}$

    - key – find default swap premium $c (*actual dollars*) such that PV CF from selling premium = PV CF from buying




- **Credit Linked Notes** ~ bond w/ payoff linked to performance (default, rating, return) of another security
  - <u>key</u> – differs from credit default swap b/c less default risk re protection seller (b/c seller sends protection payment across and buyer just pays interest until the earlier of maturity / credit event occurs)

## Exchange Offers, Holdouts, Coercion

- **exchange offer** ~ one security is exchanged for another security (or package of securities) ~ cash / new debt w lower int / equity / some combo
    - RoT – always conditioned on valid tender ⩾ 51% of bonds (often more)
    - use – mitigate financial distress + change covenants
    - why needed – B/H don't meet regularly like S/H (greater dispersion means they can't meet to agree to changes)
- **holdout problem** - if all B/H agree to exchange but one, holdout D/H realizes more return (a capital gain) than those exchanging → all B/H use same rationale → engage in prisoner's dilemma → everyone then holds out
    - value to holdout – (cf notes p. 520)

| | B does not Exchange | | B exchanges | |
|---|---|---|---|---|
| | A's Pay off | B's Payoff | A's Pay off | B's Payoff |
| A <> exchange | 0.50 | 0.50 | 1.00 | X+Y |
| A = exchange | X+Y | 1.00 | 0.5 + Y/2 | 0.5 + Y/2 |

**X** = Residual assets after redeeming D
**Y** = increase in firm value from exchange offer to B/H

- 
    - solutions - provide incentive to exchange > hold out gain (increase Y) + conditioning offer on high acceptance rate + new securities = more senior + new securities = earlier maturity + limiting pro-ration in second tier of two-tiered tender offer + limiting w/d rights + soliciting exit consents + offering prepkgd bankruptcy -
- **Coercion problem** - w/ coercive exchange offer → S/H may appropriate value from B/H tho overall firm value falls
    - S/H get abnormal gains
    - ***B/H get abnormal gains !! -*** consent payments c/b large enough to compensate B/H (evid shows re importance of Δ) **+** B/H groups have emerged to defend B/H rights / counteract dispersion of B/H **+** S/H proposals often modified → shows negotiation b/w S/H & B/H

## Bankruptcy Procedures and Bonds

- **Absolute Priority Rule** – established order in which claimants are paid in bankruptcy BUT – evid ~ APR is exception
    - Equity has implicit option to delay completion of bankruptcy process → means B/H value atrophies (costs of financial distress) → B/H willing to concede issues to expedite closure (deviations from APR reflect time value of Equity option to delay)
- **United States** -
    - goal – maximizing value of entity as a going concern (for benefit of all)
    - Ch 7 liquidation → court appointed trustee ~ liquidates assets + distributes proceeds to D/H w/ residual to E/H
    - Ch 11 reorganization – S/H (DiP) restructures org ops and capital w/ D/H receiving new securities
    - consider – B/H effectively bribe S/H to end Ch. 11 process
- **Valuation of Debt w/ Ch. 11** – ***E(T) =*** max [C(T), S(T)] ***= max [C(T), max($V_T$ – F, 0)]***
    - C(T) ~ value of option to pay F at any time before T* → see equity option valuation above (??)
    - S(T) ~ value of equity at time T in absence of Ch. 11

***How value C(T)??***

- **Relevance of Code Design to Bond Value** – mgmt can use bankrupcy process to bene selves (S/H & B/H lose) + Ch.11 negotiations → deviations from APR (B/H lose) + extent of losses is influenced by context of negotiations (Ch. 11 v. Private) + Code influences distribution of value across claimant classes

- **What can D/H do??** – more collateral + less subordination of their interests + greater covenant usage (and don't give them up)

**Securitization – Creating asset backed securities via pooling and tranching**

- **process** –
  - **1) Pooling** – set up Master Trust + specified pool of assets is sold into a Master Trust
    - Master Trust ~ separate legal / bankruptcy remote entity
    - Risks to securitization – lemon risk + default risk
      - lemon risk ~ asymmetric information means that sponsor could ***adversely select*** bad assets into pool to realize windfall gain
      - default risk ~ asset CF are risky and may default
  - **2) Tranching** – Master Trust issues securities of differing seniorities against the pool
    - first loss ~ most junior security always realizes the first loss (~ retention of subordinated interest + over-collateralization + funding of reserve account + claim on excess spread)
    - expected pool loss ~ weighted avg loss of pool (given probabilities of each loss level)
    - expected first loss ~ apply prob for each loss level to min[a risk amount, loss level] and sum → % of total expected pool loss not included in expected first loss ~ ***amount of CR risk transferred***

- **Valuing unsecured debt** (excluded from ABS pool)
  - no securitization - $[F_{unsecured} * (1 + RoR_{Promised})] = Prob_{good} * F_{unsecured} * (1+R_d) + Prob_{bad} * V_{firm\ bad}$
  - with securitization -
    - $[F_{unsecured} * (1 + RoR_{Promised})] = Prob_{good} * F_{unsecured} * (1+R_d) + Prob_{bad} * [V_{pool\ bad} + (1 + RoR_{Promised})* (Cash_{available\ to\ firm\ pre\text{-}securitization} - CE)]$
    - $R_d = \dfrac{[F_{unsec.} * (1 + RoR)] - (Prob_{good} * F_{unsec.}) - (Prob_{bad} * V_{pool\ bad}) - Prob_{bad} * (1 + RoR) * (Cash - CE)}{(Prob_{good} * F_{unsecured})}$

- **Valuing securitized debt** -
  - $[F_{pool} * (1 + RoR_{Promised})] = Prob_{good} * F_{pool} * (1+R_s) + Prob_{bad} * [V_{pool\ bad} + (1 + RoR_{Promised})* CE]$
  - $R_S = \dfrac{[F_{pool} * (1 + RoR_{Promised})] - (Prob_{good} * F_{pool}) - (Prob_{bad} * V_{pool\ bad}) - Prob_{bad} * (1 + RoR_{Promised})* CE}{(Prob_{good} * F_{pool})}$
  - $R_S$ ~ promised RoR on securitization debt ~ such that tranche is priced at par
  - CE ~ cash reserve established for bene of investors in the tranche drawn from issuer's cash assets
  - GoS ~ gain on sale booked by securitizing firm value of issuer's int in transactions excess spread ~
    - flows back to issuer in good state

- **Posit** – ***firm improves CR standing w/ securitizations that are riskier than its own debt (and vice versa)***
  - Proof – calculate $R_d - R_s$ → show the CE required to make $R_d$ & $R_S$ equal

- **Collateralized Debt Obligations** (CDOs) ~ CF linked to performance of underlying collateral Debt instruments

- **Collaterlized Loan Obligations** (CLOs) ~ CF linked to to performance of a Bank's underlying collateral Debt obligations

- **Accounting issues** -
  - FAS 125 - allows firms to recognize some of equity position in securitization + assumes that firms will not support early amortization or losses (that risk is transferred)
  - Reality – firms provide voluntary support to avoid early amortizations / losses → means originator <> transfer risk off B/S
  - Result – companies often report high GAAP earnings growth + negative ops CF

## Tax Arbitrage

- D v. E key criteria -
  - judicial - intent of parties + identity b/w CR & instrument holder + security holder's participation in mgmt + issuer's ability to obtain outside funds + thinness of capital re: to debt + level of risk + formal indicia + position of obliges relative to other CR + voting power of holder + provision for fixed rate int + contingency of repayment of the obligation + source of interest payments + use of a fixed maturity date + provision for redemption by issuer + provision for redemption at option of holder + timing relative to corp formation
  - legislative (§ 385) – formal indicia (written unconditional promise to pay) + subordination to other debt + D/E ratio + convertibility into Equity + relationship b/w holdings of stock and the interest in question

- Rate arbitrage ~ high T payers investing in tax exempt securities

- Structural arbitrage - Turning debt into equity with variable payment instruments
  - contingent obligations –
  - D-E hybrids – debt obligations where some payments vary with success of issuer business
  - other hybrids - debt obligations where some payments vary with performance of some external BM
  - hybrid notes – no return during term of note + return of principal and some variable amount (based on success of firm) at maturity
  - convertibles – (LYONS) - c/b put by holder at specified time + conversion premium accretes over time to cover accreted value (reducing prob of conversion)
  - non-convertibles – (MIPS / MIDS / Trust Preferreds)
    - issued in the form of preferred securities by non-T entity that lends proceeds to affiliated company
    - lending ~ "mirror loan" that provides for CF to cover div & redemption value on Pref securities
    - deferral of payment ~ allows SLA company has deferred div on C/S
    - key – loan c/b legally enforced after deferrel period (differs from P/S)
    - benes –
      - firm issues debt & get tax deductions / recipients receive div on P/S
      - weak covenants ~ firm can arrange to allow deferral of pay (to minimize costs of financial distress)
    - cons – no DRD
  - mandatory convertibles ~ (PERCS) - P/S mandatorily convertible at specified time

- DRD arbitrage ~
  - hedged dividend capture – corp buys stock cum dividend + sells call option on stock + liquidate stock & purchase option ⩾ 46 days later OR simply wait till option expires

***why partial hedge??***

  - adjustable rate P/S (ARPS) – w/ floating div rate lower than ST int rates
    - goal ~ make as debt like as possible → preserve spread b/w Div rate and lowest of (3) treasuries w/ qtrly reset
    - effect – since DRD applies → corp investors willing to accept lower return
    - only works if → issuer can eliminate tax on investment of ARPS proceeds (~ NOLs, T/E securities, etc.)
    - cons –
      - int rate risk - mismatch b/w 13 week div reset and 46 day holding period allows int Δ and P Δ
      - credit risk – Δ in issuer credit quality not reflected in reset mechanism

  - Dutch auction rate P/S (ARPS) –
    - goal ~ adjust shortcomings of ARPS → reset = every 7 weeks in bidding session w/ both current & potential investor
- remember –
  - arbitrage is possible b/c IRC defn of fixed income security ~ precise; var income security <> precise
  - taxation of securities considers – investor's role (investor / trader / dealer) + entity type (indiv / S / C / p'ship) + income type (capital / ordinary) + income character (active / passive / portfolio)