Funding Investments

1) Introduction – Corporate Security Design

- MM theorem -
  - assumes – no taxes + earnings are perpetuities + all earnings paid out as div + only D & E issued + investment decisions are given + debt is riskless + individuals borrow at same rate as firms
  - proof – relies on arbitrage arguments
  - event studies – cf notes 54
  - in practice – many inconsistencies with what we see in real world
    - exchange offers – effect firms value despite no new cash entering the firm
    - real world frictions – taxes + costs of financial distress

- Taxes and hybrid securities – using options to alter pay-off characteristics (Trust P/S)
  - hybrid securities – blurs distinction b/w D & E → designed to exploit tax frictions re: disctinctions b/w D &E for tax
    - similar to subordinated D and preferred E
  - Trust P/S – company issues P/S through non-tax paying SPSub (cf notes 61)

- Zero Coupon Bonds / Discount Rates –
  - spot rate (y_n) = yield on a zero
    - computed recursively from valuing coupons w/ zeroes
      - **i)** find spot rate implied in Price by (final) payment
      - **ii)** find spot rate implied by intermediate payments
  - discount factor = $1 / (1 + y_n / 2)^n$ (~ claim to one dollar in n periods)
  - gross (unannualized) rate of return = F / P – 1 (where F = future payoff and P = initial investment)

- Coupon bonds
  - P_n = **periodic payments (C) [z(t,1) + z(t,2) + … z(t,n)] + final payment (F)z(t,n)**
  - P_t = **$\text{sum}_{j=1}^{n}$ (C/2) z(t,j) + Fz(t,n)**
  - think of as a series of zeroes
  - spot rate – c/b computed recursively from valuing coupons w/ zeroes
    - **i)** find spot rate implied in Price by (final) payment
    - **ii)** find spot rate implied by intermediate payments

- coupon stripping ~ unbundling coupon bonds

- Yield Conventions –
  - US Treasuries = $1 / (1 + y_n / 2)^n$ ~ semi annual compounding

- Consider –
  - replication – one way of pricing asset = generating same CF in way that can be priced
  - arbitrage – equivalent CF s/b priced the same

- RoT -
  - only if no arbitrage opp exists → zeroes and coupons imply same discount factors and spot rates

2) Bonds – Prices, Yields, and Term Structures

<u>Bond Prices and Yields</u>

- YTM –
  - of perpetuity = coupon / MV
  - of portfolio = **(i)** sum MV of all bonds + **(ii)** sum coupon & interest for each future period + **(iii)** find IRR that equates (i) & (ii)
  - assumes – reinvestment at YTM + amort of discount / premium throughout life of bond
- current yield = coupon / MV
- holding period return (realized return) – discount rate that equates price paid for Bond w/ PV (coupon + principal)
- year conventions
  - US treasuries = 365 days
  - corp = 360 days
- coupon stripping ~ unbundling coupon bonds

<u>Money Market Instruments</u>

- includes – Treasury bills, banker acceptances, commercial paper
- yield –
  - bank discount rate (BDY) = (USD discount / 100) * (360 / t days to maturity)
    - w/b lower than the true yield (not comparable to capital market yields)
  - equivalent bond yield (if maturity < 6 mos) = (365 * BDY) / (360 – (t * BDY))
- money market yield (M) ~ **(annualized interest *100) / P** ~ **[(360 / D) * (100-P) * 100] / P**
  - if have discount rate (R) → M = **(360 * R) / [360 – (R * D) / 100]**

<u>Government Bonds</u> -

- Treasury Bills – **P = 100 – (D/360) * R** [D ~ days to maturity; R ~ % discount]
  - *consist of* - non coupon securities + 12, 26, 52 week maturity + auctioned every 4 weeks + bought / sold on bond discount yield basis (~ 360 day yr)
  - true bond equivalent yield (Y) ~ **(annualized interest *100) / P**
    - < 6 mos to maturity → Y = **[(365 / D) * (100-P) * 100] / P**
    - > 6 mos to maturity → Y = $\mathbf{[(-2D/365)+ 2*[(D/365)^2 – ((2D/365)-1) * (1-1/P)]^{0.5}] / [(2D / 365)-1]}$??
    - if have discount rate (R) → Y = **(365 * R) / [360 – (R * D) / 100]**
    - always higher than discount yield

- Treasury Notes –
  - < 6 mos ~ **P = (1+C/2) / [1+Y(D/2B)]** ??
    - B = days in the current coupon period; N = number of remaining coupons)
  - > 6 mos ~ $\mathbf{P = (C/2) / (1+y/2)^{D/B} + (C/2) / (1+y/2)^{1+(D/B)} + \ldots + (C/2) / (1+y/2)^{N-2+(D/B)} + (100 + C/2) / (1+y/2)^{N-1+(D/B)}}$
  - *quote* ~ flat P ~ clean P (P') **= P- C/2 * (B-D)/B**
  - *consist of* - coupon securities + 2-10 yr maturity + int paid semi-annually

- Treasury Bonds –
  - *consist of* – coupon securities + > 10 yr maturity + same P and Y conventions as notes
  - if issued pre-1984 → are callable at par (plus AI) in last 5 yrs to maturity
  - flower bonds – can be applied at par + AI to a dededents estate taxes (trade at lower coupon and Y)

- STRIP – coupon and principal of >10 yr Bond that is traded separately (since Oct. 1984)
  - P = $100 / (1+Y/2)^{N-1+(D/B)}$ (N~ number of 6 months periods in remaining life of STRIP)

Term Structure of Interest Rates – Represented by Yield Curves + Strip Yield Curves and Forward Rates

- term structure – relationship b/w yield and maturity on securities differing only in time to maturity
- Yield ~ IRR
  - another way of expressing P
  - assumes no re-investment risk (~ all CF received can be re-invested at same Y)
- STRIP Yield ~ geometric avg of current short rate and forward rates
- spot yield curve = graphical representation of yield / maturity relationship at a specific point in time
- structure of interest rates ~ corresponds to STRIP yield curve (not same as yield curve generally)
- zero coupon yield curve – gives YTM as function of time to maturity of zero
- zero coupon discount curve – specifies discount factor for a future time (~ zero coupon bond P for given maturity)
- pure discount bond = zero coupon bonds
- on the run – most recently issued US Treasuries (all others being off the run)
- spot rate of interest – yield on an actual bond at a particular maturity

- forward rate $({}_{t+n}r_{jt})$= $\mathbf{(1+ {}_tR_{n+1})^{n+j} / (1+ {}_tR_n)^n}$

  - rate at which 2 parties agree to lend / borrow money for specified period of time in the future
  - implicit – rate required to equate future investment alternatives (bonds) having same CF but different maturities
    - AKA – expected future spot rates
  - explicit - if embodied in K

- Accrued interest ~ **["Days" elapsed / "Days" in period] * coupon interest**
  - Actual / Actual ~ US treasury
  - Actual 365 ~ Eurobonds, Euro floating rate notes ,foreign gov't bonds
  - Actual 360 ~ Eurodollar deposits, commercial paper, banker acceptance, repos, LIBOR based transactions
  - 30/360 ~ corporate bonds, US agency bonds, muni bonds, mortgages

- ***Theories of Term Structures*** –

  - Pure Expectations Theory –
    - one version - implies that expected future interest rates are embodied by computed forward rates
    - another version – says that securities of different maturities are perfect substitutes for each other
    - in other words –
      - long rates ~ avg of today's short rate and future expected rates
      - current structure of int rates ~ set by market participants' expectations of future short int rates
    - cons –
      - market efficiency assumption + failure to account for other factors
      - only exists if market has complete certainty (forward rate would have no compensation for risk)
      - investors prefer to lend short / borrowers prefer to borrow long

  - Liquidity Premium Theory (form of Market Segmentation Theory?) –
    - says – investors demand premium for holding LT bonds (~ principal is at risk if investor liquidates prior to maturity)

  - Market Segmentation Theory –
    - says – interest rate for particular maturity is determined solely by supply / demand conditions
      - different types of lenders / borrowers prefer different maturities
    - implies that – debt management policies of issuers influence term structure

  - Cox-Ingersoll-Ross – consumption bases
    - says – individuals maximize their expected utility from consuming a single good
      - individual takes optimal level of consumption + optimal proportion of wealth to invest in each production process + invests the rest at r_f (or borrows any shortfall)

  - Multi-factor Models

  - Lattice Type Models (recombining v. non-recombining)

Price Volatility, Coupon Rate and Maturity

- coupon effect – fact that the level of a price change associated with a shift in interest rates is also influenced by the coupon
  - the lower the coupon → the greater the effect of a shift in interest rates
  - the longer the maturity → the greater the effect of a shift in interest rates

- duration = **sum of [Time to each CF * (PV of each future CF / P)] = Σ of [Time to CF * ((C/ $(1+y)^t$) / P)]**

  - weighted average of times in future when CF are t/b received (~ measured in yrs)
  - measures sensitivity of Bond P to change in yield
  - addresses fact that risks change over time such that bond may not reach maturity
    - price risk ~ Bond P moves inversely with change in interest
    - re-investment risk ~ value of re-investing coupons moves positively w/ rates
  - ~ horizon over which P Risk = Re-investment Risk
  - remember - equals maturity only with zero coupon bonds
    - duration of zero coupon bond = its maturity
    - w/ constant maturity → lower coupon rate means a higher duration (?)
    - w/ constant coupon rate → longer maturity means higher duration (always for bonds selling at par or prem)
    - w/ all else constant → lower yield means higher duration
    - duration of a perpetuity = (1+y) / y
  - for premium bonds – duration increases at decreasing rate with maturity
  - for discount bonds – duration increases at decreasing rate with maturity up to long maturity and then declines
  - for perpetuity – duration = (1+r) / r (at all maturities)
  - immediately after a payment (~ int) – duration increases in saw tooth like manner
  - see et al at Van Horne p. 102
  - remember –
    - bond investment is always immunized when duration = intended holding period
    - if interest rates shift → duration of portfolio shifts as well
  - immunization (against changes in interest rates) ~ matching bond duration w/ needed CF
    - assumes – yields on all instruments are the same and change in parallel

  - McCaulay Duration **= [(1+y) / y] + [1+y+n(C/F-y)] / [C/F($(1+y)^n$ –1)+y]**
    - ~ % change in Bond P relative to % change in (1+YTM)

  - Modified Duration **= D_{Mac} / (1+y)**

  - Duration of perpetuity = **(1+y) / y**

  - Duration of a portfolio = $w_xD_X + w_YD_Y$ = [(sum of $(CF_X + CF_Y)$) / $(1+y)^t$] / (X+Y)

  - Duration for Hedging – H = $(P_{LTBond} / P_{STBond}) * (D_{LTBond} / D_{STBond}) * [(1+ Y_{STBond}) / (1+ Y_{LTBond})]$

- convexity – cf Van Horne p. 104
  - ~ second derivative of P w/ respect to yield
  - refers to "curviness" of bond's P / Y profile
  - not a problem for non-callable bonds

- immunization of bond portfolio = dedicating bond portfolio to match a pre-set stream of CF
  - price risk – risk that, with changing interest rates, bond will need to be sold at unexpected P
  - coupon re-investment risk – risk that will have to re-invest at unexpected coupon rate
  - key – risks move in opposite directions
  - solution –
    - a) set duration equal to the intended holding period
    - b) use interest rate futures market (lower transaction costs

- equilibration - coupons v. zeroes
  - coupon stripping –
    - separation of coupons from principal amounts → each coupon / principal CF become zero coupon bond
    - new bonds are placed in Trust – trust co receives payment on original bond and repays to current holders
  - STRIP – coupon strippable product (but unit size is $1000 for each part)
    - reconstitution – can rebundle parts to get whole bond back (but need a principal piece to combine with coupon STRIPs)
  - key – arbitrage should force coupon and zero coupon yield curves to bear "a precise relationship" (??)

Of Note

- geometric average – assumes that returns are re-invested (~ avg % increment in wealth over an investment horizon)
- arithmetic average – assumes that returns are paid out (~ avg return, period by period)

3) Swaps

- swap – K to exchange future CF
  - traded under a master K
  - CR risk – accounted for in master K which specifies collateral K
- swap spread = swap rate – gov't yield (quoted in basis points, using the "on the run" gov't bond of nearest maturity)
  - "on the run" – benchmark bond most recently issued and normally trading at par

- swap resets – means that the floating rate payment changes (~ determined by forward rates applicable to reference base)

- interest rate swap –
  - long in the swap ~ fixed rate payer
  - short in the swap ~ floating rate payer
  - swap curve – relationship b/w swap rates and term to maturity

  - remember
    - int ~ calculated on notional amount

- swap rate (X) ~ % payable at each date → computed through any of following methods …
  - method 1 - fixed rate paid on the same dates as the floating rate such that PV(floating rate CF) = PV(fix rate CF)
    - where → PV of fixed rate CF = PV of floating rate CF
    - where → X * [sum of$^N_{i=1}$ (z_0,k_i)] = sum of$^N_{i=1}$ [forward rate$_{ki}$ * $z(0,k_i)$]

    - where $\mathbf{X = \dfrac{\text{sum of}^N_{i=1}\,[\text{forward rate}_{ki} * z(0,k_i)]}{\text{sum of}^N_{i=1}\,(z_0,k_i)}}$ (~PV floating rate CF) / (~PV fixed rate CF)

  - method 2 – replicate floating rate CF (since we know fixed rate CF)
    - conceptually – PV (invest $1 + receive interest on $1 + re-invest $1 at reset rate + … + receive interest & principal + assume pay off previously issued zero to eliminate $1 principal)
    - conceptually – invest $1 at time 0 + re-invest it at times (t to T) + issue a zero at time 0 and redeem zero at time T
    - such that – **X for maturity T =** $\mathbf{\dfrac{[1 - z(0,T)]}{\text{sum}^T_t\,[z(0,t)]}}$ (~PV floating rate CF) / (~PV fixed rate CF)

    - where - PV of floating rate CF ~ current cost to acquire floating rate CF = **[1 – z(0,T)]**

    - key point – swap rate (X) = T year par bond yield

  - method 3 ~ **interest rate that causes the fixed rate bond to sell at par** (given the yield curve implicit in the PV)

- day convention issues -
  - money market basis ~ ACT / 360 (assuming 30 day month)
  - bond basis ~ ACT / 365 ~ ACT / ACT ~ 30 / 360 [all being equivalent in a non-leap year]

  - process – Always do in the following order …
    - **i)** convert available info to desired period for the given basis →
      - semi-annual to annual → $\mathbf{y^* = (1+y/2)^2 - 1}$
      - annual to semi-annual → $\mathbf{y^* = [(1+y)^{0.5}-1] * 2}$
    - **ii)** convert (i) to desired bond / MM basis →
      - bond to MM basis → **M = y* 360** / **365**
      - MM to bond basis → **B = y* 365** / **360**

- asset swaps – pkg allowing investor to buy fixed rate bond + hedge almost all int rate risk by swapping fixed to floating payments (but retains CR risk)
  - essentially - when investor (rather than borrower) utilizes a swap
  - asset swap spread ~ amount investor earns for bearing CR risk

- replication – asset swap buyer pays par up front + receives fixed rate bond from asset swap seller + enters int rate swap to pay seller fixed coupon equal to asset (w/ asset swap buyer receiving floating rate payments + / - agreed spread)
  - buyer bears CR risk
  - seller PoV ~ bond is sold at par + accrued interest
- break-even swap spread ~ swap spread that makes PV of all CF = 0
  - so that → **(Up front payment to buy asset in return for par) + (PV of fixed CF) –(PV of floating CF)=0**
  - so that → $(100\text{-}P) + (C * \text{sum of}^{Nfixed}{}_{i=1} [z(0,i)]) – (\text{sum of}^{Nfloat}{}_{i=1} [\Delta_i(L_{i-1,i} + A)\, z(0,i)])$
  - note – fixed and floating legs may pay w/ different frequencies

- remember -
  - swaps determine the swap yield curve (due to their liquidity and wide array of maturities) – not vice-versa

4) Options

- writer ~ party providing the option (and liable if it is exercised)
- holder ~ party w/ right
- call -
  - $V_0 = \max[0, V_a\text{-}E]$

- Pay-offs –
  - call = Max (0, S*-K)
  - put = Max (0, K-S*)

- Put-call parity ~ $\mathbf{V_{call} + Ke^{-r(T-t)} = V_{put} + V_{sh\ of\ stock}}$
  - ROT
    - $C \geqslant 0$ – value of call is never less than 0
    - $S \geqslant C$ – value of call never more than stock itself
    - $C \geqslant S - Ke^{-r(T-t)}$ – value of call never less than stock P minus PV of exercise P
    - $C (K_1) \geqslant C (K_2)$ if $K_2 > K_1$ - value of call never less than identical call w/ higher strike P
    - $K_2 - K_1 \geqslant C (K_1) - C (K_2)$ if $K_2 > K_1$ - difference in value of identical calls never > diff in strike Ps
    - $C (t_2) \geqslant C (t_1)$ if $t_2 > t_1$ - value of call never less thab value of identical call w/ shorter t to expir.

- Black Scholes -
  - $\mathbf{P_{Call} = SN(d_1) - (Ke^{-rt})\, N(d_2)}$
    - $d_1 = [\ln (S/ K) + (r_F + (\sigma^2/2))\, t] / \sigma(t^{0.5})$ (note – assumes constant σ, but may not be)
    - $d_2 = d_1 - \sigma(t^{0.5})$
    - $N(d_1), N(d_2)$ – value of cumulative normal distribution at d_1 and d_2

  - $\mathbf{P_{Put}} = -\ \mathbf{V_{sh\ of\ stock}} + \mathbf{V_{call} + Ke^{-r(T-t)}}$
  - V_0 ~ function of ST int rate + time to expiration + variance of RoR on underlying asset (NOT expected return on underlying asset)
  - RoT
    - Call P increases w/ expiration date
    - Call P is decreasing function of exercise P
    - Call P is increasing function of volatility of underlying asset
    - Call P is increasing function of risk free rate

- Binomial Option Pricing Model –
  - Pricing Option (against stock share) ~ $\mathbf{P_{Call} = \Delta S – B}$ (S ~ P of sh of stock)
    - $\Delta = [C_U – C_D] / [S_U – S_D]$ (number of shares of stock bought to replicate the call)
    - $B = [C_U S_D – C_D S_U] / [(S_U – S_D) (1+r)]$ (B ~ amount borrowed – necessary to replicate exercise P)

    - $C_U = \Delta S_U – B (1+r)$
    - $C_D = \Delta S_D – B (1+r)$

  - Pricing Option (against Value of firm w/ debt) ~ $\mathbf{E = \Delta V_{firm} – B}$ (E ~ Value of equity)
    - $\Delta = [E_U – E_D] / [V_U – V_D]$ (number of shares of stock bought to replicate the call)
    - $B = [E_U V_D – E_D V_U] / [(V_U – V_D) (1+r)]$ (B ~ amount borrowed – necessary to replicate exercise P)

- E_U ~ max(V_U – F, 0) = **ΔV_U – (1+r)B**
- E_D ~ max(V_D – F, 0) = **ΔV_D – (1+r)B**

- Pricing Debt ~ D **= N(E+D) – B = NV_{firm} – B =** **[N / (1- N)] V_{firm} - [1 / (1- N)] B**
  - N = **[D_U – D_D] / [V_U – V_D]** (N ~ Δ)
  - B = **[D_UV_D – D_DV_U]** / **[(V_U-V_D) * (1+r)]** (B = amt borrowed)
  - D_U = min(V_U,F,0)
  - D_D = min(V_D,F,0)
  - V_{firm} = E + D
  - essentially says → risky debt CF ~ replicated by buying [N / (1-N)] V_{firm} and lending [1 / (1- N)] * B

- Inputs –
  - **i)** stock returns = log normally distributed (skewed right and never less than -100%) – r_t = ln (P_{t+1} / P_t)
  - **ii)** volatility →
    - **1)** daily log price relative (r_t) = ln (P_{t+1} / P_t)
    - **2)** est. annual variance of r_t ~ σ^2_{annual} = $(250)^{0.5}\sigma^2_{daily}$ [250 ~ typical trading days / yr]
  - **iii)** up factor = exp $[\sigma(T/n)^{0.5}]$ (~ n = number of intervals that T is split into = BS if n ~ infinity)
  - **iv)** down factor = 1 / (up-factor)

- remember -
  - for multi-period → C_U & C_D ~ additive across periods

- hedging with options
  - hedge ratio (h) = option Δ = Δ Option P / Δ Underlying Asset P
    - Δ of long position = h (~ short amount of options)
    - Δ of short position = - h
    - Δ of call option = N(d1)
    - Δ of put option = N(d1) - 1
  - delta neutrality – requires that – **100 Δ + N = 0** (where N = # shs bought; 100 used b/c 100 options in typical K)
    - portfolio ~ insensitive to small changes in P of underlying assets
  - if perfectly hedged → should earn r_f
  - BUT → if price changes → hedge ratio changes
  - in B/S → Δ of option = partial derivative of option P w/ respect to asset P
  - con – Δ = derived from model that ignores transaction costs + assumes continuous trading → requires continuous rebalancing

- Option theory and Corporate Securities -
  - remember –
    - underlying asset for corp securities = total firm value (~D + E ~ NOT observable)
    - owners of corp securities receive payouts from the corp securities
    - issuers of securities have power to alter firm decisions re investment, dividend, financing policy
  - **EQUITY** as call option on firm assets (~ B/S)
    - K = face value of debt
    - value of stock (S_t) = **V_t N(d_1) – F N(d_2) e^{-rft}** (where V ~ value of firm assets; F ~ Debt Face)
      - N(d_1) = **[ln(V/F) + (r_f + 0.5σ^2) t] / σ $(t)^{0.5}$**
      - N(d_2) = **d_1 – σ $(t)^{0.5}$**
    - S_t increases as ~ F increases; T_{debt} – t increases; σ^2 of V increases; r_f increases; value of firm assets increases
  - **DEBT** valuation (Merton Model)
    - Value of Call = Value of (Put + Underlying Asset) – PV of exercise P
    - Value of Underlying Asset = Value of Call + [PV (F) – Value of Put] [] = Value of Debt
    - For Debt →
      - underlying asset ~ firm's assets
      - equity ~ call option
      - debt ~ risk free loan and giving insurance in form of "protective" put option
      - Exercise P ~ F
      - Value of Firm's Assets = Value of Equity + [PV (F) – Put Value] [] = value of risky debt
      - Value of Risky Debt = Value of Default free bond – Value of Put
        - Value of Put ~ discount demanded by B/H for default risk
        - ~ B/H position = risk free claim on firm + Put option written for bene of Equity

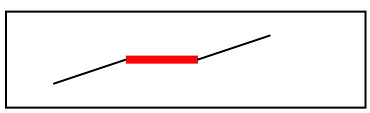
        - if $D_f = D_r$ → must mean value of Put = zero (Equity will exercise their call)
- Value of Debt ~ V_t – **Equity** ~ $V_t - C(V_t;F, \sigma, t)$
    - at payoff – $D^* = Min[F, V^*]$
    - prior to maturity – $D_t \sim PV(F)$ – **Value of put** = $F e^{-rt} - P(V,F,t)$
- Problem – Equity may change non-systematic risk of firm (asset substitution)

- Credit Spread ~ $r - r_f$ (b/c in bond pricing → more common to examine promised YTM than Ps)
    - key determinant ~ σ^2 ~ unknown
    - Value of default free promise to pay in the future (D_f) = $F e^{-r_f t}$
    - Value of risky promise to pay in the future (D_r) = $F e^{-rt}$
    - YTM (r) = $\ln [F/D]/t$
        - per Garbade – $r = r_f - \{\ln[L^{-1} N(d_1) + N(d_2)]\} / t$ (where $L = D_f / V$)

- Junior Debt – $D_J = C(V; F_S) - C(V, F_S + F_J)$
    - $V = D_S + D_J + S$
    - $D_S = V - C(V;F_S)$
    - $S = C(V; F_S + F_J)$
    - $V = [V - C(V;F_S)] + D_J + C(V; F_S + F_J)$
    - note – D_J may be an increasing function of T-t and volatility (D_J can act like equity)

- remember –
    - if value of Risky Debt ~ value of risk free debt → little risk that Equity will put firm to B/H
    - corporate securities <> true derivatives b/c Value of underlying assets is under control of mgmt

- debt options –
    - futures options – give right to buy / sell from / to a writer a futures K at specified price at any time during option period
        - have thrived (while spot options have not) due to accy complexities
    - interest rate cap – c/b manufactured by purchasing a Put
        - additional cost of borrowing is offset by increased value of Put
        - overall cap c/b broken into separate periods (~ caplets)
    - collar – exists if borrower accepts a cap and a floor (separate periods involve floorlets)
        - pro – much cheaper than a straight cap
    - corridor – combination of two caps (buy cap at lower interest rate strike level + sell cap at higher strike level)
        - interest cost = interest on straight floating rate loan – difference b/w two strike interest rates
        - effect – only partially protects buyer (within middle level band) → no protection outside band



    - bene of options – loss is limited to premium paid (no such bound for futures K)

    - valuing debt options –
        - Black Scholes –
            - variance = key (can't assume constant b/c they approach and eventually mature – i.e., upside is bounded by face value)
            - American options – debt options can be exercised at any point up to maturity
        - binomial lattice type model – again dependent on how accurately volatility is mapped

- Yield curve options – cf van Horne p. 177

- convertible securities = bond or P/S c/b converted at option of holder to C/S
    - essentially = straight debt + option value = Convertible Debt Value
        - remember – value of debt and option are both effected by volatility in corp's CF
    - bene - corp issues security at lower yield
    - value = calculate bond value floor (i.e., if value of C/S falls)

    - conversion ratio (or price) = ratio of exchange b/w convertible and the C/S
    - conversion value = conversion ratio * Market P of a share of C/S
    - conversion premium = price differential b/w non-convertible debt and convertible debt
    - premium over conversion value = MP of convertible bond in excess of its conversion value (due to downside protection inherent in the bond value floor)

- premium over bond value = MP of convertible bond in excess of its bond value (assuming C/S has value)

Remember

- options are zero sum game – can win only if someone else loses
- hedging – change in value of option (from simply holding it) offsets change in value on long position

5) Bond Covenants

Covenants

- positive covenants – require positive action
- negative covenants – prohibit some action
- defeasance – voiding / annulment of a K (or a provision w/i a K achieving this affect)
- sinking fund – fund that accumulates to pay off corporate debt at some point in the future
- event risk – likelihood that the rating of a bond will drop due to an event, such as the taking on of additional debt or a recapitalization by a company
- negative pledge clause – covenant in a bond agreement whereby the borrower agrees not to pledge any assets if such pledging would result in less security for the agreement's bondholders. (also called covenant of equal coverage.

- goal – prevent asset substitution (~ changing ops, spending on neg. R&D)
- effect –
  - limits extent to which mgmt can continually increase risk over time (~ cuts short lower branch of tree) by forcing liquidation
- remember –
  - impossible to predict / prevent everything → difficult to limit asset substitution and allow flexibility to mgmt
  - if asset substitution is costless → S/H will always engage in it
  - B/H want to include covenant so long as L > F (below F, the incentive to include falls b/c going concern value lost)

Down and Out (Knockout / Barrier) Options

- identical to European call + cancellation of K if V < some preset boundary + liquidated damages upon K cancellation
- cf – notes 231 - prof says this is optional

Evidence of effectiveness – exists for strong covenants (not for weak covenants) → cf. notes 233

Strong covenants (SPPC) –

- quick triggers – activated if single S/H gains control of some % of shs
- slow triggers – require higher control %
- effect – S/H adversely affected + B/H no affect + mgmt benefits

Asset Substitution

- limited by bond covenants
- will always occur if transaction costs related to substitution are zero

6) Callable Corporate Bonds

- callable bond – bond w/ embedded option giving issuer right to redeem bond at specified P

- why include call option?
  - interest rate risk ~ allows the option to redeem if rates fall …
    - conceptually – if rates fall → redeem old bond and replace with new (possibly straight) debt which could be issued at a premium (at a one time gain) for the same coupon
    - BUT investors know this and demand higher int rate (~PV of expected m/b same as PV of non-callable bond)
    - so → issuer w/b forced to issue callable at w/ a coupon that makes them indifferent w/ issuing straight debt
  - underinvestment – where existing debt matures after an investment opp (assumes int rates have fallen?)
    - some of CF go to B/H → equity holders require a higher RoR
    - discounting CF from investment opp at higher rate can make project NPV negative (meaning firm passes)
    - underinvestment results
    - solution ~ callable debt → corp can refinance to avoid this problem
  - flexibility – bonds may have covenants preventing firm from taking the investment opp
    - callable debt allows corp to refinance and mitigate the covenant limits
    - BUT – bonds often callable at / above par (so calling only makes sense if already trading at / above par)

        - solution – set call premium at % above discounted value of bond coupons & principal, using r_f

- Proof of relationships → identify transactions allowing arbitrageur to realize positive CF today w/o change in future CF
    - **if $P_C < P_{NC} - C$ →**
        - purchase callable bond for P_C
        - purchase option on bond for C (~ allows to cover short position)
        - sell short non-callable bond for P_{NC} (~ for more than $P_C + C$)
    - **if $P_C > P_{NC} - C$ →**
        - purchase non-callable bond for P_{NC}
        - purchase option on bond for C (~ allows me to call if company calls bond form me)
        - sell short callable bond for P_C (~ for more than $P_{NC} + C$)

    - key point → arrange such that revenue on initial transaction is a riskless arbitrage
        - but option does not trade → so would have to buy / sell portfolios of other securities that replicate the option

- Pricing -
    - Black Scholes – does not work b/c no closed form solution if can be called at any time for varying strike P (K*)
        - **if bond called** –
            - Bond Payoff ~ K_t
            - Stock Payoff ~ $V_t - K_t$
        - **if bond not called ($V_t > K_t$)** –
            - Bond Payoff ~ K
            - Stock Payoff ~ V* - K
        - **if bond not called ($V_t \leqslant K_t$)** –
            - Bond Payoff ~ V*
            - Stock Payoff ~ 0
        - **Valuation in terms of American Option** –
            - Bond value ~ V-C(V; K_t; K) …. Call option struck at K_t at time t and K at maturity
            - Stock value ~ C(V; K_t; K)

    - Binomial Option Pricing Model –
        - Value of D -
            - at interim ~ Max **[λ V, Min(call P, D+C)]**
            - at maturity ~ **Max [λ V, Min (V, D+C)]**
        - assume Bond called if value + accrued interest > call P

- today -
    - less calls issued – mispricing of call option has gone away –
    - due to emergence of "swaption" market (which made callable bonds easier to price)
    - options on interest rate swaps c/b tailored to replicate terms of embedded options
        - option on interest rate ~ option to exchange fixed rate bond for floating rate bond

- Replication with Swaption ~ X yr fixed rate bond w/ call after X-t yrs → purchase X yr non-callable bond + sell swap for right to receive fixed for t yrs starting in yr (X-t)
    - cf notes 28

- remember
    - Value of Callable Bond = Value of Identical Non-callable Bond – Value of Call Option → $\mathbf{P_C = P_{NC} - C}$
    - Value of Call Option = PV of one time gain from issuer exercising the call option and replacing it with straight debt paying same coupon
    - Bond prices (changing from D to D_U or D_D) are driven by changes in **i)** r_f and **ii)** credit risk

7) Convertible Bonds

- convertible security – security that, at holder's option, may be exchanged for another security w/ different characteristics
    - callable bonds ~ almost always convertible
- conversion P – paying using the convertible's face value
- call protection – initial period of time when company will not call bonds
- investment value – value of a convertible w/o conversion option

- PRIDES ~ Preferred Redeemable Increased Dividend Equity Security ~ mandatory convertible preferred share (at maturity)
    - w/ dividend yield > underlying common stock (to compensate for lower participation in U/S and no limit on D/S)
- floorless convertible – guarantees investors conversion value (~ conv. P = current trading P of stock)

- Pricing –
    - Black Scholes –
        - conversion ratio (r) ~ # shs of C/S obtained upon surrender of the convertible debenture
        - conversion P = **Face Value / conversion ration = K / r** (K = face value)
        - new shs issued on 100% conversion = **Mr** (M = # convertibles outstanding)
        - dilution factor (λ) ~ equity % owned by converting B/H = **Mr / (N + Mr)** (N = # shs O/S pre-conversion)
            - anti-dilution clauses – typically protect conv. bonds from dilution if shs issued at below market
        - conversion RoT – convert only if ~ **$\lambda V^* > K$**

        - payoffs at maturity –
            - **if $V^* \leqslant K$** -
                - Bond Payoff ~ V^*
                - Stock Payoff ~ 0
            - **if $K < V^* < K/\lambda$** -
                - Bond Payoff ~ K
                - Stock Payoff ~ $V^* - K$
            - **if $K/\lambda < V^*$** -
                - Bond Payoff ~ λV^*
                - Stock Payoff ~ $(1-\lambda)V^*$
            - **Valuation in terms of American Option** –
                - Bond value ~ **$V - C(V; K) + \lambda\, C(V; K/\lambda)$** [equity~ $-C(V; K)$; debt call on CS ~ $\lambda\, C(V; K/\lambda)$]
                - Stock value ~ **$C(V; K) - \lambda\, C(V; K/\lambda)$**

        - Value of convertible Bond (B_C) = **$B(V,K) + \lambda\, C(V; K/\lambda)$** [$B(V,K)$ ~ straight debt]
            - at maturity → min [V^*, max(K, λV^*)]

    - Binomial Option Pricing Model –
        - w/ call and convertability → find value of convertability by solving for D
        - Value of D
            - at interim ~ Max **[λ V, Min(call P, D+C)]**
            - at maturity ~ **Max [λ V, Min (V, D+C)]**

- who issues – cos w/ debt below investment grade (young, low prob of surviving, stronger incentive to asset substitute, less info avail., etc.)

- benes
    - ~ allows investors to circumvent investment / equity holding restrictions
    - ~ provides another asset class to facilitate diversification
    - ~ signaling – mgmt is optimistic re: future stock P prospects (~ depends on asymmetric info) – cf proof on p. 333
    - ~ reduces risk of asset substitution (by mgmt) – b/c also benefits convert holders
    - ~ reduces affect of volatility (risk) of firm → downside to debt is offset by upside to equity claim
        - not b/c conv. debt is cheaper – (BUT may allow "synergistic" access to financing over straight debt for very high risk cos)

- Propositions –

    - **1)** conversion of convertible security – never convert before maturity if stock pays no dividends and markets are perfect (no TC, taxes, etc)
        - proof (p. 327) – consider value of two portfolios (convert or no convert) in all possible states of world

    - **2)** conversion of callable convertible security – never convert except at maturity or call if stock pays no dividends and markets are perfect
        - proof (p. 328) - consider value of two portfolios (convert or no convert) in all possible states of world
        - option always worth more alive than dead

    - **3)** calling a convertible security – call when conversion value = call P + ε (mgmt should force as soon as possible b/c contra says that is in best interest of convert owners to delay as long as possible)

- optimal call policy ~ call IFF current V_{firm} > V#
  - V#t satisfies V#t ⩽ K(t) / λ for all t
  - K(t) ~ call P + accrued int
  - proof (p. 329) – key point is that convertible is worth more when option is alive
  - maximize V_{equity} while minimizing $V_{convertible}$
- limits to optimal call policy – usually B/H get 30 days notice + transaction costs for new issuance (if debt is redeemed rather than converted) + some argue is "unfair" to deprive B/H of conversion privilege + mgmt concern for diluting UEPS (since mgmt comp often tied to EPS) + market signaling (if mgmt usually clears debt before bad events by forcing conversion)

- remember -
  - Convertibles <> cheap source of financing → higher than straight and subordinated debt b/c conversion feature
  - convertible = straight debt + warrant
  - issuer can induce conversion → raise dividend

8) Credit Risk: Bond Ratings and Spreads

- credit risk – likelihood that obligor will not honor a promised debt payment (chance of default by an issuer)
- relative risk – refers to ordering of risks
- absolute risk – refers to ability of rating t/b guide to E(loss)
- bond rating – indicator assigned by a rating agency (~ ordinal measure of likelihood of bond making promised payments)
  - default is ranked (not quantified)
  - different bonds of same company may have different rankings
  - ***Moody's*** – Aaa – Caa
  - ***S&P*** / ***Fitch*** – AAA – D
  - ***D&P*** – AAA – DD
- junk bond – bond w/ at least one rating below BBB- level from either S&P or Moody's

- rating drift = ***(# upgrades - # downgrades) / # rated issues*** (~ summarizes overall incr. / decr. in CR quality as % of one letter grade)

- rating activity = ***(# upgrades + # downgrades) / # rated issues*** (~ shows fraction of rated issues ~re-rated)

- bond spread ~ (YTM for a corp bond) – (YTM of equiv. maturity (or duration) Treasury or LIBOR rate bond)
  - spread ~ compensation for risk of default ~ viewed as indicator of default (increase means > possibility of default)
  - actual value as indicator → aggregate factors across firms are more important than firm specific factors (p. 399)

- Why do firms pay for ratings?
  - rated firm can release inside information w/o disclosing to public
  - ratings agency is doing a service to investors → rated firm is trying to help investors invest in their securities

- causality – is there info in ratings?
  - empirical studies on yield spreads show causality from ratings to Ps of bonds
  - study of effect of bond downgrades shows negative correlation w/ bond Ps (but is it the downgrade or the related negative event?)
  - study of effect of Moody's providing more info in ratings → information in ratings changes ~ re default risk (and largely diversifiable)

- remember –
  - rating is based on default (not expected recovery once default occurs)
  - correlation exists b/w bond rating and risk of default (lower rating → shorter time to default)
  - probability of default for a given rating varies throughout the business cycle

9) Pricing Risky Bonds – using bond ratings

- probability method –
  - c ~ coupon rate
  - δ ~ recovery rate ~ % recovery of promised int + principal promised on a bond that defaults
    - some bankruptcy courts allow D/H to claim par; others allow claim par + c
  - p ~ credit risk ~ probability of default (bond issuer will be unable to make timely payments of promised int)
  - z ~ discount factor ~ $[1 / (1+r)]$
  - λ ~ loss rate ~ $1-\delta$
  - q ~ probability of an uptick in value for BOPM purposes
  - default risk structure of interest rates ~ relationship b/w time to maturity and credit spread ~ how spot yield on risky zero coupon bond varies w/ time to maturity

  - expected future loss = $\boldsymbol{100p(1+c)(1-\delta)}$

  - expected rate of return $(1+r) = \{(1-p)(1+c) + p[\delta(1+c)]\}$
    - assumes bond selling at par and investors are risk neutral

  - pricing equation -
    - → $100 = z[p\delta + 1-p] * \$100\ (1+c)$
    - → $1+r = (1+c)\ [p\delta + 1-p] = (1+c)[1-p(1-\delta)] = (1+c)(1-p\lambda)$
    - → $c = [(1+r)/ (1-p\lambda)] -1 = (r+p\lambda) / (1 - p\lambda)$
    - → $p = (c-r) / \lambda(1+c)$

  - credit spread $(c-r)$ =
    - $= [(r+p\lambda) / (1 - p\lambda)] - r$
    - $= [(r+p\lambda) - r(1-p\lambda)] / (1 - p\lambda)]$
    - $= [p\lambda + rp\lambda] / (1- p\lambda)$
    - $= p\lambda(1+r) / (1- p\lambda)$

  - cons – neither p nor δ are known

- BNOP –
  - Value$_{call}$ (C) $= [pC_U + (1-p)\ C_D] / (1+r)$ → says $C = (Prob_{uptick} * V_{call\ on\ uptick} + Prob_{downtick} * V_{call\ downtick}) / (1+r)$
    - p ~ true probability of security P moving up / down IFF people are really risk neutral
    - ***key point*** – said 3 different ways …
      - BNOPM solution for value of call = value of call in risk neutral economy
      - attitudes re risk <> matter for option P
      - ***valuation based on no arbitrage b/w replicating portfolio and the security is identical to risk neutral valuation***

  - causality – runs from returns on underlying asset at each binomial outcome to probability (p) – not vice versa
    - → risk neutral probabilities (p) ~ probabilities that make expected return of an asset = r_f
    - → p must solve $(p)u + (1-p)\ d = 1+ r$ → $p = \dfrac{(1+r-d)}{(u-d)}$
      - u ~ (1 + RoR on underlying asset in up node)
      - d ~ (1 + RoR on underlying asset in down node)
      - - w/ RoR calced over period that up / down move can occur

  - this type of risk neutral valuation – c/b applied wherever (i) replicating portfolio c/b constructed OR (ii) risk neutral (p) c/b inferred from traded securities OR (iii) investors risk ~ neutral re their risk prefs

- Transition probabilities and matrices -

  - recovery rates (δ) ~ amount B/H receives at default (determined mainly by – bond seniority + rating at default)
    - varies over time (due to effect of business cycle, change in bankr. laws, etc.)

	Recovery Rate for Given Year of Maturity*								
Debt Senior.	**74 - 91**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**
Sr. Secured	59.02	76.04	64.30	87.78	76.29	74.63	33.72	60.81	67.13
Sr. Unsecured	46.56	69.97	33.65	67.39	44.57	43.26	35.29	45.50	46.53
Sr. Subord.	45.69	32.87	28.84	47.73	39.85	40.27	26.47	33.68	32.10

Subordinated	39.32	33.93	31.14	37.75	35.03	25.95	23.39	16.58	26.44
Jr. Subord.	-	-	-	-	55.00	17.50	18.86	3.63	17.95
Wgted Avg.	48.83	40.29	35.76	72.57	44.29	39.11	28.17	32.65	39.22

* of corp defaults in given yr, this is avg P of bonds that traded w/i 2 weeks of announced default (~ market expectation)

- transitional probabilities – prob that bond of particular rating will move to another rating over some indicated time
- Markov processes – evolution of transition probabilities
  - ***$P_{risky\ bond}$ = $P_{riskless\ bond}$ * [Par – (chance will default(Q) X expected loss)]***
    - $D^i_{(t,T)} = D^f(t,T) * [1 – Q_{(t<T)} (1-\delta)]$
  - pro – volatility of firm is irrelevant (so can use for complicated capital structures) + can incorporate changing interest rates
  - cons – assumes transition probs + recovery rates are stable over time (probably not true) + default occurs only at maturity (works w/ only a single period (?) discount bond in capital structure)
  - remember
    - once a bond enters default it stays there → any emergence from bankruptcy is effectively new bonds
    - $Q_{i,j}$ ~ transition probability matrix (the area w/ the probabilities) – i = row; j = column
    - Q^x ~ matrix Q multiplied by itself (x) times
    - probability of moving to default over 2 yrs ~ sum of all possible products of combos that end in default
  - avg one year transition prob (1981 – 1991)

	Rating at End of Year								
Initial Rating	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**	**NR**
AAA	0.875	0.095	0.008	0.002	0.003	-	-	-	0.018
AA	0.008	0.879	0.073	0.010	0.003	0.003	-	-	0.025
A	0.001	0.028	0.861	0.063	0.010	0.004	-	0.001	0.032
BBB	0.001	0.004	0.062	0.800	0.061	0.015	0.002	0.004	0.055
BB	-	0.002	0.007	0.065	0.720	0.094	0.012	0.022	0.097
B	-	0.002	0.003	0.006	0.045	0.72	0.038	0.060	0.127
CCC	-	-	0.010	0.010	0.018	0.067	0.573	0.204	0.118

- remember –
  - use V-tree b/c B/S really only works for ST (rarely use pure discount debt + σ^2 is time variant + mgmt may asset substitute
  - we assume (to price debt) …
    - people are risk neutral (this makes easier to use historic results to generate probs) – a stretch
    - term structures <> correlated w/ recessions (more reasonable)
  - as λ or p go to zero → credit risk coupon rate (c) approaches riskless rate and (c-r) goes to zero
  - can infer prob of default if assume all (c-r) was due to default risk
  - ***BOPM solution for C_{value} ~ value of call in risk neutral economy (b/c only time that p = q)***
    - ***→ attitudes toward risk ~ irrelevant for option pricing***
    - ***→ valuation based on no arbitrage b/w replicating portfolio and security are identical to risk neutral valuation***
  - while credit risk varies as move through binomial tree → we only care re chance of default

10) Credit Default Swaps (as a Credit Derivative)

- credit derivative ~ security w/ payoff contingent on a [credit rating (rare), default performance (more), or return performance] of underlying reference asset
- credit default swap ~ agreement where one party purchases insurance against default of specified reference credit / security / group of securities

- underlying reference asset – some other security, such as bond or loan, on which a credit derivative is made
- protection seller ~ long in credit exposure
  - receives premium in exchange for potential payment to protection buyer in event of credit event
  - periodic premiums continue until credit event occurs
- protection buyer ~ short in (underlying) credit exposure
- credit event ~ event of default (e.g., failure to scheduled interest / principal, bankruptcy, insolvency, receivership, acceleration, restructuring,m repudiation, et al)

- premium ~ swap quote ~ ***paid semi-annually / quarterly in arrears on actual / 360 basis***
  - ~ expressed in bps per annum over swap tenor
- cash flows -
  - INCEPTION - no exchange of principal

  - PERIODIC payment to protection seller ***= notional amt * bid-ask amt (in bps) * year convention adjustment***

  - DEFAULT payout to protection buyer ~ depends on the type of instrument

    - cash settled payout → ***notional amount * max [(100-$P_{default}$),0] / 100***

    - binary payout → ***notional amount * max [(100-x),0] / 100*** (x ~ set in advance, ignoring events)

    - physically settled → ***protection buyer has right to put defaulted security to protection seller at par***
      - more common than cash settled (applies to all single name credit default swaps)
      - risk = short squeeze
- uses -
  - hedging an exposure ~ credit default swap ~ avoids liquidity issues, waiting for exposure to roll over, etc.
  - creating synthetic floating rate notes (assets) ~
    - **i)** write default protection on a bond
    - **ii)** invest remaining of $100 principal in near MM equivalent asset
    - **iii)** investor earns R per period on $100 + c per period on credit default swap
    - **iv)** on default → use MM investment to cover swap payment
    - bene - synthetic asset c/b cheaper than the market equivalent
  - creating synthetic short position – swap ~ allows longer short positions b/c synthetically creates short sale of the reference asset

- key features -
  - only credit risk is transferred (does not include an interest rate component)
  - no investment of principal (no money changes hands at swap inception)
  - default swap payments ~ subject to default swap risk of swap counterparty

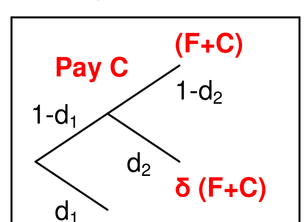
- pricing –
  - replication ("no arbitrage") – ***NOT POSSIBLE***
    - assume - reference note is par floating rate note (FRN_{risky}) + no payment of accrued credit swap premium at default + can shor the reference note today for current MV and then buy back note on date of CR event + $FRN_{riskless}$ exists (floating rate R_t at date t) w/ coupon pays of R_t + S (~ S being fixed spread) + default is physically settled
    - replicating process –
      - **i)** short sell FRN_{risky} (~ receiving $100)
      - **ii)** invest receipts in FRN $_{riskless}$ ~ held to earlier of (maturity, credit event)
      - **iii)** pay R_{risky} + S ; receive R_t
      - **iv-a)** if no credit event → both FRN mature at par and no net CF at termination
      - **iv-b)** if credit event → both …

        - **1)** liquidates portfolio at next coupon date after event
          - → buy FRN_{risky} in market + deliver against short position (~ thru repo K)

        - **2)** collects $MV_{risk\ free\ FRN} - MV_{risky\ FRN}$ =D = 100 - $P_{default}$ (at coupon date, $MV_{risk\ free\ FRN}$ ~ par)

    - cons – too many simplifying assumptions + assumption that wide range of corp bonds c/b shorted for LT <> valid + reference asset may not be traded (so not possible to cover short position)

    - conclusion – replication does not work, so P by arbitrage is not possible

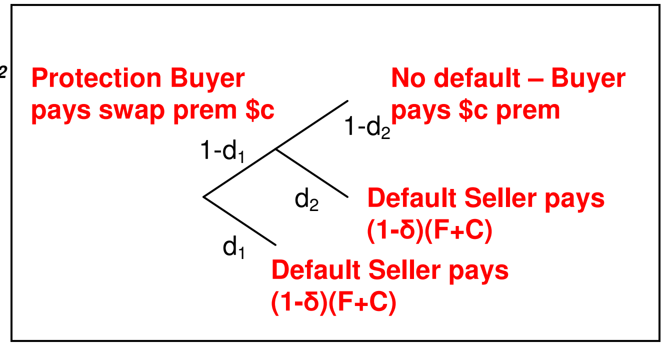


- <u>model based - bond pricing</u> – $P = sum^{n}_{t=1} \left(\frac{s_t C + s_{t-1} d_t \delta (F+C)}{(1+r)^t} + \frac{s_n F}{(1+r)^n} \right)$

$$P = s_1 C z_1 + s_0 d_1 \delta (F+C) z_1 + s_2 (F+C) z_2 + s_1 d_2 \delta (F+C) z_2$$

  - p_t ~ cumulative prob of defaulting up to just prior to date t
  - d_t ~ prob of defaulting in next period given survival to date t
  - s_t ~ prob of surviving (not defaulting) to date t ~ $1 - p_t$ ~ $(1-d_1)(1-d_n)$ [~ EXCEPT → $s_1 = (1-d_1)$]
  - requires – risk neutrality

- <u>model based Credit Default Swap</u> –
  - $c(s_1 z_1 + s_2 z_2) = s_0 d_1 (1-\delta)(F+C) z_1 + s_1 d_2 (1-\delta)(F+C) z_2$
    (to seller) (to buyer)
  - $c = \frac{s_0 d_1 (1-\delta)(F+C) z_1 + s_1 d_2 (1-\delta)(F+C) z_2}{(s_1 z_1 + s_2 z_2)}$
  - key – find default swap premium $c (*actual dollars*) such that PV CF from selling premium = PV CF from buying



- Credit Linked Notes ~ bond w/ payoff linked to performance (default, rating, return) of another security
  - <u>key</u> – differs from credit default swap b/c less default risk re protection seller (b/c seller sends protection payment across and buyer just pays interest until the earlier of maturity / credit event occurs)

- remember -
  - post default → protection buyer pays no more premiums
  - payout events are infrequent but cause severe losses
  - short squeeze – possible any time have physical settlement (too much is sold so P of underlying asset explodes if everyone forced to deliver at same time) → more possible in commodities markets (e.g., Korean bond)

11) Exchange Offers, Holdouts, Coercion

- exchange offer ~ one security is exchanged for another security (or package of securities) ~ cash / new debt w lower int / equity / some combo
  - <u>RoT</u> – always conditioned on valid tender ⩾ 51% of bonds (often more)
  - <u>use</u> – mitigate financial distress + change covenants
  - <u>why needed</u> – B/H don't meet regularly like S/H (greater dispersion means they can't meet to agree to changes)
  - <u>problem1</u> ~ ***holdout gains*** – if all B/H agree to exchange but one, holdout realizes more return (a capital gain) than those exchanging
    - all B/H ~ rationale → engage in prisoner's delimma → will then holdout
  - <u>solutions</u>
    - provide incentive to exchange > hold out gain –
      - ensure that benefit to tendering B/H (~ increase in firm value from tendering?) ⩾ than holdout gain
      - such that bene to B/H of exchange > hold out, irrespective of what other B/H do

	B does not Exchange		**B exchanges**	
	<u>A's Pay off</u>	<u>B's Payoff</u>	<u>A's Pay off</u>	<u>B's Payoff</u>
A <> exchange	0.50	0.50	1.00	X+Y
A = exchange	X+Y	1.00	0.5 + Y/2	0.5 + Y/2

X = Resid value of D to B/H
Y = increase in firm value from exchange offer to B/H

    - conditioning offer on high acceptance rate – ensures illiquidity for remaining holdouts

        - new securities = more senior – holdouts lose priority
        - new securities = earlier maturity – holdouts are paid last
        - limiting pro-ration – two-tiered tender offer guarantees pro-ration to those who tender before deadline
        - limiting w/d rights – prevents people from changing mind once tendered
        - soliciting exit consents – holdouts lose covenant protection w/o any comp
        - offering prepkgd bankruptcy -

    - problem2 ~ w/ coercive exchange offer → S/H may appropriate value from B/H tho overall firm value falls

    - empirical study (p. 524) –
        - shows that - S/H generally use consent solicitations to loosen covenants that limit expropriation of B/H value
        - abnormal bond return (a) = ***$(P_{+1} / P_{-1}) - [P(y_{-1} + \Delta y_t) / P_{-1}]$***
            - P_{+1} = Bond P after event date
            - P_{-1} = Bond P before event date
            - Δy_T = change in matched maturity Treasury yield from before event date to after
            - Δs = change in credit spread b/w bonds of similar ratings and Treasuries over event date
            - P(y) = standard P function for given YTM (y)
        - results –
            - S/H get abnormal gains
            - ***B/H get abnormal gains !!***
                - consent payments c/b large enough to compensate B/H (evid shows re importance of Δ)
                - B/H groups have emerged to defend B/H rights / counteract dispersion of B/H
                - S/H proposals often modified → shows negotiation b/w S/H & B/H
    - remember
        - tendering firm often offers premium over market

- consent solicitation ~ issuer offers to pay B/H to change covenant (~ NO exchange of securities involved)
    - key pt – approved changes affect all B/H even if not consenting
    - Trust Indenture Act of 1939
        - Δ int / principle ~ all B/H must approve
        - other Δ ~ less B/H may approve (~ look to indenture agreement)

- exit consent ~ consent solicitation that is combined w/ an exchange offer
    - issuer may require consent to participate in exchange

- coercive exchange offers ~ results from techniques employed to limit holdout gains in exchange offer
    - allows S/H to expropriate value from B/H

- tender offer ~ security is exchanged for cash

- hold-out – S/H or D/H who rejects exchange offer and retains their claim to par

- financial distress ~
    - costs ~ EE retention probs + difficult finding trade CR + consumers not wanting to buy product + increases in agency / monitoring costs + loss of financial flexibility + loss of sales from weakened assurance of delivery + inability to undertake O/W profitable ventures + costly violations of covenants

    - solved by – exchange offer / new investment / acquisition / bankruptcy / liquidation

12) Bankruptcy Procedures and Bonds

- bankruptcy – CR can assert K rights to take control of corp assets ~ legal process for re-allocation of corp control rights
    - caused by - insufficient CF caused by exogenous (poor economy) and endogenous (mis-management → debt serves to control S/H / mgmt) factors
    - goal – create liquidity w/i firm to preserve firm + balance exogenous / endogenous factors & not entrench SH or mgmt
- Bankruptcy Code ~ determines parties negotiating rights in bankruptcy
- pre-packaged bankruptcy ~ reorg plan agreed to by most parties before entering bankruptcy (to receive formal approval)

- Absolute Priority Rule – established order in which claimants are paid in bankruptcy

- BUT – evid shows that APR is exception (not rule) → means B/H lose
- Equity has implicit option to delay completion of bankruptcy process → means B/H value atrophies (costs of financial distress) → B/H willing to concede issues to expedite closure (deviations from APR reflect time value of Equity option to delay)
- Private restructurings – return more value to all classes of D/H
  - empirical evid (cf p. 564) → private restructuring favors E/H over D/H (w/ greater return than if APR applied)

- United States -
  - goal – maximizing value of entity as a going concern (for benefit of all)
  - Ch 7 liquidation → court appointed trustee ~ liquidates assets + distributes proceeds to D/H w/ residual to E/H
  - Ch 11 reorganization – S/H (DiP) restructures org ops and capital w/ D/H receiving new securities
    - DiP gets – retains control of business + gets 120 day (extendable by court) automatic stay to propose plan + seeks DiP financing (subordinates existing CR) + can propose cramdown
      - Automatic Stay – CR cannot take action to enforce claim by exercising control over assets
    - CR gets – CR committee is set up to monitor business ops (protect against mismanagement)
    - Test of any plan – vote by impaired CR (majority of number + 2/3 of value) and S/H
    - cramdown – court can impose a plan if any objecting class of CR receives ⩾ what w/h received in Ch. 7
  - consider –
    - if bankruptcy process were costless → possibility of bankruptcy <> affect firm's capital structure (?) or P of liab. b/c any loss to S/H would be captured by D/H through bankruptcy process (note Texaco / Pennzoil losses)
    - going concern Value > liquidation value (or simply liquidate b/c restructuring in bankruptcy makes no sense)
    - easier for firm to seek voluntary Ch. 11 (post 1978) + firm does not have t/b insolvent
    - empirical evidence re 1978 changes to Ch. 11 – S/H lose more (~ everything) + B/H lose more + mgmt wins (b/c survival rate of firms increased from 74.5% to 83.3%)
    - B/H effectively bribe S/H to end Ch. 11 process

- United Kingdom -
  - goal – repayment of CR
  - applies if – firm is insolvent
  - process – firms immediately come under management of insolvency practitioner representing interests of CR
    - liquidation –
    - receivership – CR appoints receiver (w/ personal liability) to run firm & decide to liquidate / re-organize + equity gets zero
    - voluntary reconstruction –
    - administration – court appoints administrator (no personal liability)
  - benes – lower costs (less negotiation)
  - cons – premature liquidation + less flexibility (must liquidate if liquid value > going concern value)

- debt valuation w/ Ch. 11 –
  - assume – no one knows true value of firm + F of pure discount debt is due at time T + mgmt has exclusive right to delay payment of debt until T* > T + judge imposes APR at time T* IFF firm not reorganized by then + after an initial time (t') to arrange their affairs, CR may buyout S/H option to delay
  - value – ***E(T) =*** max [C(T), S(T)] ***= max [C(T), max(V – F,)]***
    - C(T) ~ value of option to pay F at any time before T*
    - S(T) ~ value of equity at time T in absence of Ch. 11

- Impact of Code Design on Bond Value – mgmt uses bank process to bene selves (S/H & B/H lose) + Ch.11 negotiations → deviations from APR (B/H lose) + extent of losses is influenced by context of negotiations (Ch. 11 v. Private) + Code influences distribution of value across claimant classes

13) Securitization – Creating asset backed securities via pooling and tranching

- securitization – act of selling cash flows separate from the assets that generate them
  - creates tradable / liquid debt securities from illiquid assets

- **process** –
  - **1) Pooling** – set up Master Trust + specified pool of assets is sold into a Master Trust
    - Master Trust ~ separate legal / bankruptcy remote entity
    - Asset Backed Security (ABS) characteristics – pooled assets have no legal / mgmt influence connection w/ originator / servicer + pool ~ large & homogenous & diversified → predictable CF + CF supplemented w/ credit enhancement + excess spread
      - excess spread – excess yield on underlying assets over interest and service fees paid on ABS pool ~ measure of financial health of the pool
      - credit enhancement –
        - ~ cash collateral account (CCA) ~ separate funded account to cover shortfalls, may include deposits of excess spread
        - ~ collateral invested account (CIA) ~ (privately placed) residual equity security that funds the pool
    - Risks to securitization –
      - lemon risk ~ asymmetric information means that sponsor could ***adversely select*** bad assets into pool to realize windfall gain
        - assumes - some assets are worth less + asymmetric info (*sponsor knows which assets are bad + investors cannot tell which assets are good / bad prior to investment*) + sponsor of good asset has no credible signal to give to market + moral hazard (buyer cannot trust sponsor) + adverse selection (only sponsor of bad asset has incentive to pool) + externality effect (returns to good car sale accrue to bad car seller)
        - effect ~ illiquidity (market inefficiency / failure) → b/c investor pays no more than the E(value) of assets ~ avg P (good & bad) + sponsor of good asset refuses to pool good assets at a loss + only bad assets are placed in pool + investors will only pay the P of bad assets
        - solution – sponsor credibly commits to selecting random portfolio + credit enhancement
      - default risk ~ asset CF are risky and may default
    - process – originator pools assets + originator standardizes assets (~ maturity, rate, amount, etc.) to enhance CF predictability + credit is enhanced by third party guarantees (for a fee) + SPV created (to hold pool remotely from originator) + tranching (below)
    - e.g., Credit Card Receivables - three stages ~ revolving period (principal collections used to buy new assets generated generated in designated accounts) + controlled amortization period (principal collected used to repay investors) + early amortization (if triggering event → tap credit enhancer)
    - pros – lower cost of financial distress (b/c bankruptcy remote vehicle) + improve liquidity of illiquid assets
  - **2) Tranching** – Master Trust issues securities of differing seniorities against the pool
    - first loss ~ most junior security always realizes the first loss (~ retention of subordinated interest + over-collateralization + funding of reserve account + claim on excess spread)
    - expected pool loss ~ weighted avg loss of pool (given probabilities of each loss level)
    - expected first loss ~ apply prob for each loss level to min[a risk amount, loss level] and sum → % of total expected pool loss not included in expected first loss ~ ***amount of CR risk transferred***

- **Valuing unsecured debt** (excluded from ABS pool)
  - goal - given an E(RoR) to all assets / D → find R_d that provides same return to unsecured D
  - no securitization - $[F_{unsecured} * (1 + RoR_{Promised})] = Prob_{good} * F_{unsecured} * (1+R_d) + Prob_{bad} * V_{firm\ bad}$
  - with securitization -
    - $[F_{unsecured} * (1 + RoR_{Promised})] = Prob_{good} * F_{unsecured} * (1+R_d) + Prob_{bad} * [V_{pool\ bad} + (1 + RoR_{Promised}) * (Cash_{available\ to\ firm\ pre\text{-}securitization} - CE)]$
    - $$R_d = \frac{[F_{unsec.} * (1 + RoR)] - (Prob_{good} * F_{unsec.}) - (Prob_{bad} * V_{pool\ bad}) - Prob_{bad} * (1 + RoR) * (Cash - CE)}{(Prob_{good} * F_{unsecured})}$$
  - note –
    - cash on balance sheet reduces r_d below RoR on assets (b/c cash is not risky)

- **Valuing securitized debt** -
  - $[F_{pool} * (1 + RoR_{Promised})] = Prob_{good} * F_{pool} * (1+R_s) + Prob_{bad} * [V_{pool\ bad} + (1 + RoR_{Promised}) * CE]$
  - $$R_S = \frac{[F_{pool} * (1 + RoR_{Promised})] - (Prob_{good} * F_{pool}) - (Prob_{bad} * V_{pool\ bad}) - Prob_{bad} * (1 + RoR_{Promised}) * CE}{(Prob_{good} * F_{pool})}$$
  - R_S ~ promised RoR on securitization debt ~ such that tranche is priced at par
  - CE ~ cash reserve established for bene of investors in the tranche drawn from issuer's cash assets
  - GoS ~ gain on sale booked by securitizing firm value of issuer's int in transactions excess spread ~
    - flows back to issuer in good state

- **Posit** – ***firm improves CR standing w/ securitization that are riskier than its own debt (and vice versa)***
  - Proof – calculate $R_d - R_s$ → show the CE required to make equal

- **Accounting issues** -
  - FAS 125 - allows firms to recognize some of equity position in securitization + assumes that firms will not support early amortization or losses (that risk is transferred)
  - Reality – firms provide voluntary support to avoid early amortizations / losses → means originator <> transfer risk off B/S
  - Key – on B/S debt ~ tax advantage to issuer (no bene for off B/S debt that still liable for)
  - Analysis –
    - adjusted leverage = Adjusted D / adjusted E
      - Adjusted D ~ B/S debt + debt issued by securitization trust where sponsor retains CR risk
      - Adjusted E ~
        - ~ tangible common E – after tax amount of retained interest int in securitizations
        - ~ reverse gains form securitizations and add back excess spread as income
    - adjusted EBITDA = (EBITDA – GoS) / interest expense
      - b/c GoS <> available to pay interest expense
  - Result – companies often report high GAAP earnings growth + negative CFO

- **Collateralized Debt Obligations** (CDOs) ~ CF linked to performance of underlying collateral Debt instruments
  - actively managed CDO pool – SPV manager alters pool over time according to specified criteria
    - bene – allows manager (~ hedge fund) to participate in and raise money
  - arbitrage CBO – through market inefficiency → CBO P's may remain stable as corp spreads widen

- **Collaterlized Loan Obligations** (CLOs) ~ CF linked to to performance of a Bank's underlying collateral Debt obligations
  - bene – allow banks to reduce capital requirements by transferring risk off B/S + faster than other securitizations (b/c using CR derivatives allows to dispense w/ SPV)
  - synthetic CLO – cf page 738
    - bene – if investors can't create themselves + agencies maybe mis-rating things + asset mgr (???)

- **remember** -
  - first loss position limits risk transferred through securitization
  - GoS impacts issuers D/E ratio (shifting risk should impact WACC)

- R_S varies inversely w/ size of CE; R_d varies positively w/ size of CE (goes up as CE goes up)
- outright sale ~ special case of securitization w/ zero CR enhancement (impact on unsecured creditos depends on riskiness of assets sold ~ consider disposal of cash to retire D)

14) Tax Arbitrage

- tax arbitrage – act of minimizing taxes
- tax shelter – structure that minimizes taxes

- D v. E key criteria -
  - judicial - intent of parties + identity b/w CR & instrument holder + security holder's participation in mgmt + issuer's ability to obtain outside funds + thinness of capital re: to debt + level of risk + formal indicia + position of obliges relative to other CR + voting power of holder + provision for fixed rate int + contingency of repayment of the obligation + source of interest payments + use of a fixed maturity date + provision for redemption by issuer + provision for redemption at option of holder + timing relative to corp formation
  - legislative (§ 385) – formal indicia (written unconditional promise to pay) + subordination to other debt + D/E ratio + convertibility into Equity + relationship b/w holdings of stock and the interest in question

- Rate arbitrage ~ high T payers investing in tax exempt securities

- Structural arbitrage - Turning debt into equity

  - variable payment instruments –
    - contingent obligations –
    - D-E hybrids – debt obligations where some payments vary with success of issuer business
    - other hybrids - debt obligations where some payments vary with performance of some external BM
    - hybrid notes – no return during term of note + return of principal and some variable amount (based on success of firm) at maturity
  - hybrids –
    - convertibles – (LYONS)
      - c/b put by holder at specified time + conversion premium accretes over time to cover accreted value (reducing prob of conversion)
    - non-convertibles – (MIPS / MIDS / Trust Preferreds)
      - issued in the form of preferred securities by non-T entity that lends proceeds to affiliated company
      - lending ~ "mirror loan" that provides for CF to cover div & redemption value on Pref securities
      - deferral of payment ~ allows SLA company has deferred div on C/S
      - key – loan c/b legally enforced after deferrel period (differs from P/S)
      - benes –
        - firm issues debt & get tax deductions / recipients receive div on P/S
        - weak covenants ~ firm can arrange to allow deferral of pay (to minimize costs of financial distress)
      - cons – no DRD
    - mandatory convertibles ~ (PERCS) - P/S mandatorily convertible at specified time

- DRD arbitrage ~
  - hedged dividend capture – corp buys stock cum dividend + sells call option on stock + liquidate stock & purchase option ⩾ 46 days later OR simply wait till option expires
    - ***why partial hedge??***
  - adjustable rate P/S (ARPS) – w/ floating div rate lower than ST int rates
    - goal ~ make as debt like as possible → preserve spread b/w Div rate and lowest of (3) treasuries w/ qtrly reset
    - effect – since DRD applies → corp investors willing to accept lower return
    - only works if → issuer can eliminate tax on investment of ARPS proceeds (~ NOLs, T/E securities, etc.)
    - cons –
      - int rate risk - mismatch b/w 13 week div reset and 46 day holding period allows int Δ and P Δ
      - credit risk – Δ in issuer credit quality not reflected in reset mechanism
  - Dutch auction rate P/S (ARPS) –
    - goal ~ adjust shortcomings of ARPS → reset = every 7 weeks in bidding session w/ both current & potential investor

- consider – not all T payers incur same T rate → DRD causes corps to value div differently than individuals
    - find % realized on transaction and annualize to compare to other investments

- remember –
    - arbitrage is possible b/c IRC defn of fixed income security ~ precise; var income security <> precise
    - taxation of securities considers – investor's role (investor / trader / dealer) + entity type (indiv / S / C / p'ship) + income type (capital / ordinary) + income character (active / passive / portfolio)

App. A Option Theory (Review)

- Defn
    - call – right to buy stock → all upside & total downside protection
    - put – right to sell stock → protection from P rise
    - premium – price paid by buyer to acquire option
    - exercise P ~ strike P – price paid to acquire underlying security
    - expiration date – last date option c/b exercised
    - European Option – exercised only at time of expiration
    - American Option – exercised at any time prior to / at expiration
    - Expected Return (r_{tM}) = $(P_t - P_{t-1} + D_t) / P_{t-1}$ → annualized $r_t = 12 * r_t$
    - geometric mean –
        - $P_t = P_0 (1+r)^t$
        - continuously compounded → $P_t = P_0 e^{rt}$
            - $r_t = \ln (P_t / P_{t-1})$
    - trading days – assume 252 in US

- Calculation
    - **C** – exercise P
    - **$P_{S,T}$** – stock P at time to expiration (**T**)
    - **e** - 2.7183
    - **N(d$_1$), N(d$_2$)** – value of cumulative normal distribution at d_1 and d_2

    - Payoff Matrix -

	$P_S < C$	**$P_S > C$**	**In other Words**
Call Option (Long)	0	P_S - C	max (0, P_S – C)
Put Option (Long)	(C – P_S)	0	max (0, C – P_S)

- Put-Call Parity – construct portfolio to create synthetic "riskless" security

    - e.g.,
        - **A)** construct portfolio
            - **i)** purchase share (long in stock)
            - **ii)** buy a put option (long)
            - **iii)** write a call option (short)

        - **B)** calculate payoffs –

	$P_S < C$	**$P_S > C$**	**In other Words**
i) Call Option (Short)	0	- (P_S – C)	max (0, P_S – C)
ii) Put Option (Long)	(C – P_S)	0	max (0, C – P_S)
iii) Stock	P_S	P_S	P_S
Net Position ($P_{S,T} + P_{P,T} + P_{C,T}$)	+ C	+ C	+C

- S/H v. B/H ~ equity writes a call option (short) to D/H

	$V_L < D$	$V_L > D$	In other Words
B/H	V_L	D	max (V_L, D)
S/H	0	V_L – D	max (0, V_L - D)

- Binomial option pricing model – determining Option Premium by forcing ROI = r_F
  - says – if we know r_F → we can determine Call Premium (since (i) can construct riskless security through put / call parity + (ii) force ROI to equal r_F)
  - how
    - **i)** identify possible states of the world
    - **ii)** find combination of securities (put, call, stock) that create risk free return (i.e., return the same amount regardless of the state of the world that finally occurs)
    - **iii)** determine value of securities in various states of world
    - **iv)** determine P_{option}
      - **a)** r_F = [Risk free payoff / ($P_{S,0}$ – P_{Call})] – 1
      - **b)** P_{option} = $P_{S,0}$ – [Risk free payoff / (1+r_F)]
  - e.g.,

	Depression	Prosperity	In other Words
Value of Stock	$P_{S,D}$	$P_{S,P}$	will not be same
Value of Option	0	- ($P_{S,P}$ – C)	focus on payoff to buyer
Total Payoff	$P_{S,D}$	$P_{S,D}$ - ($P_{S,P}$ – C)	s/b same value

  - gamma (δ) – number of call options needed t/b written to fully hedge a long stock position
    - fully hedged – goal is to guarantee a payment equal in both states of D & P
    - e.g.,

	Depression	Prosperity	In other Words
Value of Stock	$P_{S,0}$	$P_{S,0}$	will not be same
Value of Option	0	- δ ($P_{S,P}$ – C)	focus on payoff to buyer
Total Payoff	$P_{S,D}$	$P_{S,D}$ – δ($P_{S,P}$ – C)	s/b same value

    - solve for gamma – choose δ such that $P_{S,D}$ = $P_{S,D}$ - δ ($P_{S,P}$ – C)
      - **δ = [$P_{S,D}$ - $P_{S,P}$ / ($P_{S,P}$ – C)]** ~ ratio of range of P_S to range of option payoffs
    - solve for option P
      - ~ P_{Call} = 1/δ [$P_{S,0}$ – [$P_{S,D}$ / (1+r_F)] (~ where PS,D = risk free payout)
  - European Call (1 period binomial)
    - priced as –
      - **a)** fully hedged long position in stock +
      - **b)** writing δ call options w/ strike P of C
    - valuation =
      - **i)** P_{Call} = 1/δ [$P_{S,0}$ – [$P_{S,D}$ / (1+r_F)]
      - **ii)** 1/δ ($P_{S,0}$) - ($P_{S,D}$ / δ C) [C/ (1+r_F)]
      - **iii)** binomial distribution = HR_1 ($P_{S,0}$) – HR_2 [C / (1+r_F)]
        - > where HR_1 = 1 / δ and HR_2 = ($P_{S,D}$ / δ C)

- Black Scholes Option Pricing Model
  - **a)** generally –
    - assumptions – continuous trading in stocks & options + continuously compounded constant r_F + σ of stock return is constant + no dividends + continuous RoR of stocks are normally distributed (~ stock P = log normally distrib) + P moves as random walk
    - ignores – Dividends + American Options
    - European Call -

- $P_{Call} = N(d_1)\, P_{S,0} - N(d_2)\, (P_{S,0}\, e^{-rFt})C$
- $P_{Call} = HR_1\, P_{S,0} - HR_2(P_{S,0}\, e^{-rFt})C$ ~ similar to binomial distribution
- where
  - > $d_1 = [\ln (P_{S,0} / C) + (r_F + (\sigma^2/2))\, t] / \sigma(t^{0.5})$ (note – assumes constant σ, but may not be)
  - > $d_2 = d_1 - \sigma(t^{0.5})$

– implied volatility – compensates for fact that σ <> observable → found w/ iterative approach (no closed form inversion of problem) by - either
- **a)** estimate from time series data of stock returns
- **b)** estimate volatility by option premiums observed in market
- **c)** pure plays

– variables in Option Pricing

Traditional Option	Observable?	Capital Project	
i) C - Strike Price	Y	Amt expended (X)	
ii) σ^2 - Risk of underlying asset	N	σ^2 of project returns	
iii) r_F	Y	r_F	
iv) $P_{S,0}$ – Current P of underlying asset	Y	Value of asset built (S)	
v) t – time to maturity	Y	time can wait w/o losing opp	

➢ **b)** Dividends -
- – American Call (no div) → no incentive to exercise before maturity (i.e., same as European)
- – American Put (w or w/o div)→ may be incentive to exercise when put is deep in money
- – American Call (small Div) ~ not enough to early exercise → adjust BS to reflect that not entitled to div
  - $P^*_{S,0} = P^*_{S,0}$ – Sum $e^{-rFt}D_t$
- – American Call (large Div) → may be exercised early → see special valuation models if div are known
- – Uncertain Div → not possible to form risk free hedge portfolios → Call options cannot be valued w/o considering risk preferences

➢ **c)** Warrants –
- – note – dilution effect, but firm gets cash
- – calculation -
  - N = O/S shares
  - M = O/S European warrants (one sh per warrant (t) yrs later at price (C))
  - $W = P_{warrant} = P_{Call} * N / (M+N)$ – accounts for dilution
  - $P_{S,0}* = P_{S,0} + (M/N)W_I$

  - **i)** identify N, M, t, σ, r_F, C, $P_{S,0}$ (known or estimated)
  - **ii)** guess at W_I [e.g., $W_I = N / (M+N) * P_{Call}$ (~ of equivalent warrant)]
  - **iii)** calc $N(d_1)$ & $N(d_2)$
  - **v)** $P_{call} = P_{S,0}* N(d_1) - N(d_2)\, Ce^{-rFT}$
  - **vi)** $W_E = N / (N+M) * P_{Call}$
  - **vii)** go to step (ii) until $W_B = W_E$

- Real Options
  - ➢ Background
    - – consider –
      - downsizing (a) is investing against future losses + (b) closes off future opps (forfeits option to resume π ops)
      - opportunity to invest in project is like a call option (investing kills the option's value)
      - in practice → people discount at rates > OCoC
      - substantial value of most cos = options to invest / grow
      - investment = irreversible when
        - > specific to industry (everyone sees as worthless) or company (~sunk costs)
        - > gov't regulation
        - > differences in corp culture
      - option value = EV w/ option – EV w/o option
      - generally better to wait for uncertainty to be resolved
      - acts that create options → more valuable than NPV indicates
      - scale v. flexibility – latter may be nmore valuable in face of uncertainty

- where large sunk costs exists → closing = opportunity cost b/c lose intangibles
- cons of NPV =
  - Assumptions → investment is irreversible + now or never proposition + ignores value in creating options through current investment + in practice, WACC is simply used as disc rate
  - leads to missing valuable ops + missing the timing
- key question = when to exercise option

- option theory of investing – as concept progresses toward implementation → options disappear and value falls(hurdle rate rises)

- Capital Projects
  - valuation methods -
    - **a)** DCF – applies if project cannot be delayed (t = 0) or has no variance ($\sigma = 0$)
      - NPV_q = PV (CF) / PV (Capex) → RoT ~ invest if greater than 1
    - **b)** option – when project c/b delayed
      - NPVq – still matters, but is influenced by riskiness of project
      - **option value** = look up % X underlying Asset Value (S) for NPVq and cumulative variance (σ ($t^{0.5}$))
  - cumulative variance = σ^2 X t (~ variance of project times time remaining on project)
    - measures how much things could change before time runs out
  - RoT - capital project s/b pursued if PV(CF) > PV (Capex) at maturity of option

IF	NPV	NPVq	$(\sigma^2 t)^{0.5}$	then
i)	< 0	<1	zero	never exercise
ii)	< 0	<1	low	doubtful prospects
iii)	< 0	<1	med / high	less promising – require active development
iv)	< 0	>1	high	very promising b/c high variance
v)	> 0	>1	low / medium	wait if possible
vi)	> 0	> 1	zero	exercise now – will not change

  - consider
    - we can value European Call w/ only cumulative variance and NPV_q
    - option w/ σ or t = 0 have no variance and c/b valued w/ DCF
    - simplify complex projects ~ sequence of serially dependent projects
      - **i)** identify main uncertainty
      - **ii)** find upper / lower bounds
    - estimating σ
      - **i)** guess – systematic B and total risk are positively correlated in large samples of operating assets
      - **ii)** estimate – historical data / implied volatility from quoted option P (note – equity returns are levered and more volatile than underlying asset returns)
      - **iii)** simulate σ - Monte Carlo
      - **iv)** locate project w/i "tomato garden" plot – if know whether cumulative variance is high / low

- remember
  - lack of symmetry b/c can't go to negative infinity
  - P_0 ~ C (1 / 1+r_F) t ~ Ce^{-rFt} (if continuous compounding)
  - E(r) return on fully hedged position = r_F → $[P_{S,D} / (P_{S,0} - \delta P_{Call})] - 1 = r_F$
  - Loan Guarantee = Risky Bond + Put on loan
    - $P_{Put} = F(V_0, r_F, T, D, \sigma_V)$
      - where – σ_V^2 = Var (D + E) = Var(D) + Var(E) + 2Cov (D+E)
  - must adjust NPV for opportunity cost of option
  - exercising options kills option value