
NOTE

- merger requires –

I. Terminal Value Calculation

- Perpetuity Cash Flow – either growing / stable → use $[CF_{t+1} / (r-g)^t]$ ~ (Gordon Growth Model)
- Multiple of BV – multiply the ending <u>total capital</u> (in Terminal year) times some representative MV / BV multiple (current multiple for the company may be a good approximation unless expected to change over time)
- Multiple of earnings - multiply <u>earnings</u> (in Terminal year) times some representative P/E ratio (current multiple for the company may be a good approximation unless expected to change over time)
- liquidation value - calculate liquidation value in Terminal year; liquidation value might be some % of BV

II. Cash Flow Issues

- EBIT
  - EBIT = CFD + CFE + taxes
- Capital Cash Flows – Value of entire firm → CF to both D & E (including tax deductible interest)
  - CCF ~ <u>after tax cash flows</u> available to all security holder (D & E)
  - CCF
    - = Operating CF – tax
    - = ECF + DCF
    - = FCF + ITS
  - OCF = CF from operations + CF from Investments <u>excluding</u> CF to Capital holders (i.e., interest)
  - discount rate ~pre-tax r_A (i.e., expected return on assets)
    - $r_A = r_F + (B_{Asset})$ (risk premium)
- Equity Cash Flows – Value of Equity → CF available to S/H after payments to D/H
  - Debt Cash Flows ~ payments of interest and principal to D/H
  - valued as a perpetuity using r_E
  - ECF
    - = CCF – DCF
    - = OCF – tax – int – debt repay + debt proceeds issued
    - = $(EBIT-r_D D)(1-T) + Dep - \Delta NWC - CAPX - \Delta D$

- discount rate - r_E (~higher risk b/c subordinate to DCF → higher rate than r_A)
  - $r_E = r_F + B_{Equity}$ (risk premium)
  - $B_{Levered\ Equity} = B_{Asset}\ /\ (E/V) = B_{Asset} * (V/E)$

- Debt Cash Flows -
  - DCF
    - $= r_D D + \Delta D$

- Free Cash Flows – Value of entire firm → value whole firm (but deals with taxes outside of CF by including in discount rate)
  - FCF = OCF – tax
  - valued as a perpetuity using WACC
  - discount rate ~ WACC = $r_E(E/V) + r_D(D/V)(1\text{-}t)$
  - note – r_A = WACC + ITS = WACC + $t^* (D/V) * r_D$

- Summary –

If D/V = constant → DCF			
CF	Discount Rate	Value	Comment
UFCF	r_A	V_U	special case where D= 0 (~ D/V = 0)
UFCF	WACC	V_L	special case where D/V is fixed ▪ $NPV_U = V_U\text{-}C_O$ ▪ $NPV_L = V_L\text{-}C_O$
CFD	r_D	D	NPV = E – equity investment (??)
CFE	r_E	E	$r_E = r_A + (r_A – r_D)(1\text{-}T)(D/E)$
If D/V <> constant → APV			$V_L = V_U$ + DVITS ▪ DVITS = $TV_L – TV_U$ ▪ DVTS = $(I \times T) / (1+r_D)^T$
Return on Equity Method			~ simply find IRR on CFE (if above hurdle rate → accept) ▪ BUT if D/V is not constant → no IRR exists ▪ BUT if NPV is positive at hurdle rate → IRR m/b > hurdle rate
Decision tree			better to use this method when default is imminent ▪ ~ each period → calc probability will default next period
IRR			▪ does not adjust for size ▪ poor for mutually exclusive projects ▪ $IRR_{UFCF} \geq r_{WACC}$ ▪ $IRR_{CFE} \geq r_E$

III. Discount Rate Estimation

- CAPM
  - note
    - use b/c there is no better alt
    - estimate for single stock is very noisy

- **CAPM applies to any asset**
  - discount rate of <u>any</u> CF_i → $k_i = r_f + B[E(r_m)-r_f]$
- return on asset ~ sum of all cash flows + change in MV
- B – measures magnification or attenuation of market moves on the value of asset i
  - measures <u>proportionality</u> of change in asset value relative to market change
- R^2 ~ measures tightness with which market changes affect changes in asset value
  - measures <u>fraction</u> of change in asset value explained by market
- r_f ~ return on a portfolio that guarantees no variation in wealth
- market – all possible investment alts
  - caveats –
    - i) most n'th level worrying is unnecessary due to other sources of errors in process +
    - ii) accurate benchmark ~ impossible & unnecessary SLA actual index is exposed to all major factors generally affecting the economy
  - index used ~
    - in practice = S&P 500 (but is unrepresentative and skewed to US large caps)
    - better = Wilshire 5000 or Morgan Stanley World Index
    -
- implicitly relies on arbitrage theory – if asset is overpriced, investors will leave until P becomes better aligned

- discount rate -
  - ~ <u>required return</u> needed to induce investors to take a position in an asset
  - ~ <u>expected return</u> of an investor from investing in an asset

- estimating B
  - not observable BUT may be found thru regression
  - typically – monthly returns over 5 yr period (trade-off b/w noise in ST and more observations over LT)
    - key – can use other firms to estimate B
  - S/E affects reliability of coefficient estimate (generally not reported)
  - reject an estimate either -
    - **i)** B estimate is noisy (~ if change horizon, B estimate changes substantially)
    - **ii)** SE is so large as to make B unusable (i.e., 2 SE from μ)
      - BUT – SE of average of n estimates is divided by $n^{0.5}$ IFF avg n estimates of the same true B

- estimating the Estimated Market Risk Premium (EMRP)
  - ~ upward bias in the security return if security has typical exposure to systematic risk (B = 1)
  - <u>main issues</u> –
    - **i)** more observations v. timely data → we want more observations (not timeliness b/c we must drive down SE + can't use other firms to estimate MRP (b/c there is only one market))
    - **ii)** how compute avg MR –
      - typically – S&P 500 from 1926 to present (but avg varies with period chosen)
      - problem – lots of noise (impossible to tell if EMRP varies over time) + can't say when market s/b hi / low b/c no external references
      - avg – arithmetic mean (not geometric) b/c it better approximates the true (one yr) μ return (due to Law of Large numbers)
    - **iii)** how to define r_f - reference is US treasury over the period of the CF (despite lots of variation (annual return σ) with LT bonds)
- CAPM cons
  - **a)** no empirical relationship b/w B & return +
  - **b)** other factors do better +
  - **c)** difficult to justify projecting discount rate from many stocks onto one
  - **d)** typical variability of B is large (and unknown)

- Multi-factor models – SKIPPED

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- remember
  - for illiquid assets → MV may be stale (and return may be harder to calculate)
  - CAPM ignores R^2 b/c diversification eliminates the fraction of change in asset value not determined by market changes

- CAPM is less a theory of expected returns and more of a theory of expected return premiums
- P multiple better than DCF at tracking trends in EMRP
- CAPM is theoretically graceful but not empirically sound
- ALWAYS consider stat variability of a B estimate in addition to its estimated coefficient

IV. Valuation Methods

- APV -
  - **i)** Value firm as if all equity financed (i.e., discount at r_A)
  - **ii)** Value ITS
    - discount rate should equal the riskiness of CF being valued
      - real risk is ~ that EBIT < interest
      - i.e., there is less risk of this than risk associated w/ EBIT itself (if firm is prudently leveraged)
      - thus – OK to use lower discount rate than that on equity
      - common convention ~ use pre-tax interest rate
  - **iii)** Value other "side effects associated w/ financing –
    - e.g., Tax loss carryforwards
    - discount at rate that reflects risk (~ pre-tax rate on debt)
- Equity Cash Flow Valuation –
  - in general
    - use for - valuing highly levered equity claims on operating assets
    - alternative to option pricing (which is preferable but may be impractical)
    - bene – while it gives a bias result (similar to other methods), at least we know it underestimates Equity Value
  - theory – owners of highly leveraged equity have call on asset
    - challenge ~ complex capital structure means that equity holders have sequence of nested options to exercise call any time one of myriad of debt obligations comes due
    - w/ lower leverage → less important to value owners option to walk away from debt
  - calculation -
    - ECF = CCF – DCF = OCF – tax – (int – debt repay + debt proceeds issued) [see above]

V. Optimal Capital Structure

- Generally – interest deductibility means that firm may realize lower R_A and still meet its required R_E and R_D
- But –
  - **A)** Agency costs of debt – CEO may pursue C/F negative projects
  - **B)** Default risk –
    - calc value at given risk level and find probability of not being able to meet interest commitments
    - direct costs – legal / accy fees + lack of secondary market for assets
    - indirect costs – customers don't want to deal (stigma) + COI b/w B/H & S/H + lack of control in bankruptcy
  - **C)** limited π tax → means finite beneficial debt load
  - **D)** debt covenants likely will impede operations (especially as Debt load grows) ~ limiting value transfers
  - **E)** riskiness of business → higher earnings volatility increases probability of insufficient int coverage
  - **F)** asset type – some assets are less liquid / have lower liquidation value than others (further impairing ability to cover interest in a pinch)

VI. Capital Budgeting

- measure of worth
  - payback period -
    - essentially ~ time for stream of CF to equal original cash outlay
    - cons – ignores post payback CF + ignores TVM

- Avg RoR –
  - essentially - proceeds divided by yrs during which received + then divide by original investment
  - cons – ignores TVM + ignores duration of proceeds (~ bias towards ST investments)
- PV – essentially – today's value of money t/b received in future

- NPV –
  - process – choose discount rate + calc PV of cash receipts + calc PV of cash outflows + compare both PVs
- IRR –
  - essentially – find int rate that makes PV of cash inflows & outflows equal (~ T&E)
  - used to calculated yield to maturity of investment in gov't bonds
  - if projects = mutually exclusive → use NPV (or choose one with higher IRR if NPV not available, but beware)
  - cons – ignores total return (~ mutually exclusive projects) + multi switch b/w in/out flows = multi IRRs + possible to have no IRR (if + - +) + fails if OCoC changes over time
  - pros – allows to eval projects w/o computing CoC

- Two Key issues
  - **1)** <u>C/F</u>
    - ***include*** -
      - **i)** only relevant cash flows – incremental (~ OH) + after tax
        - ~ compare projects on apples to apples basis
      - **ii)** all incidental effects (erosion / other benes)
        - cannibalization – will someone else come in if you don't
      - **iii)** working capital needs (~ WC includes cash needed to operate company
      - **iv)** sustaining Capx
      - **v)** opportunity cost – e.g., need to invest in supporting PPE early
      - **vi)** real option value
        - at least recognize effect (positive / negative) on alts
    - ***exclude*** –
      - **i)** sunk costs
      - **ii)** allocated OH costs
      - **iii)** CF of unrelated projects
    - ***inflation*** – discount real CF at real rate (and nominal CF at nominal rate)
  - **2)** <u>discount rate</u> –
    - represents OCoC + reflects risk & TVM

- consider
  - commodity sales → P should go down w/ more production
  - sunk engineering costs → consider timing of larger project (concern = front loading)
  - terminal value – may make up significant portion of value
    - value perpetuity (but recognize Capx to maintain)

VII. Valuing an Acquisition Candidate (Kennecott)

- why acquire
  - <u>diversification</u> –
    - operating synergy → changing C/F
    - financial synergy → changing r
    - S/H can do this BUT
      - **a)** incomplete capital markets may make conglomerate more efficient (~ emerging markets)
        - size of conglomerate → gives ability to tap markets
      - **b)** mgmt talent may be scarce

- Issues
  - **i)** identify appropriate CF – any synergies?

- ignore Acquirer's tax loss – this has nothing to do with any particular Target
- **ii)** what discount rate –
  - have to look at Target and its business risk
  - also → depends on which CF you are discounting
- **iii)** what capital structure

VIII. Option Theory

- Defn
  - call – right to buy stock → all upside & total downside protection
  - put – right to sell stock → protection from P rise
  - premium – price paid by buyer to acquire option
  - exercise P ~ strike P – price paid to acquire underlying security
  - expiration date – last date option c/b exercised
  - European Option – exercised only at time of expiration
  - American Option – exercised at any time prior to / at expiration
  - Expected Return (r_{tM}) = $(P_t - P_{t-1} + D_t) / P_{t-1}$ → annualized $r_t = 12 * r_t$
  - geometric mean –
    - $P_t = P_0 (1+r)^t$
    - continuously compounded → $P_t = P_0 e^{rt}$
      - $r_t = \ln (P_t / P_{t-1})$
  - trading days – assume 252 in US

- Calculation
  - **C** – exercise P
  - **$P_{S,T}$** – stock P at time to expiration (**T**)
  - **E** - 2.7183
  - **N(d_1), N(d_2)** – value of cumulative normal distribution at d_1 and d_2

  - Payoff Matrix -

	$P_S < C$	**$P_S > C$**	**In other Words**
Call Option (Long)	0	P_S - C	max (0, P_S – C)
Put Option (Long)	(C – P_S)	0	max (0, C – P_S)

- Put-Call Parity – construct portfolio to create synthetic "riskless" security

  - e.g.,
    - **A)** construct portfolio
      - **i)** purchase share (long in stock)
      - **ii)** buy a put option (long)
      - **iii)** write a call option (short)

    - **B)** calculate payoffs –

	$P_S < C$	**$P_S > C$**	**In other Words**
i) Call Option (Short)	0	- (P_S – C)	max (0, P_S – C)
ii) Put Option (Long)	(C – P_S)	0	max (0, C – P_S)
iii) Stock	P_S	P_S	P_S
Net Position ($P_{S,T} + P_{P,T} + P_{C,T}$)	+ C	+ C	+C

- S/H v. B/H ~ equity writes a call option (short) to D/H

	$V_L < D$	**$V_L > D$**	**In other Words**
B/H	V_L	D	max (V_L, D)
S/H	0	V_L – D	max (0, V_L - D)

- Binomial option pricing model – determining Option Premium by forcing ROI = r_F
  - says – if we know r_F → we can determine Call Premium (since (i) can construct riskless security through put / call parity + (ii) force ROI to equal r_F)
  - how
    - **i)** identify possible states of the world
    - **ii)** find combination of securities (put, call, stock) that create risk free return (i.e., return the same amount regardless of the state of the world that finally occurs)
    - **iii)** determine value of securities in various states of world
    - **iv)** determine P_{option}
      - **a)** r_F = [Risk free payoff / ($P_{S,0}$ – P_{Call})] – 1
      - **b)** P_{option} = $P_{S,0}$ – [Risk free payoff / (1+r_F)]
  - e.g.,

	Depression	Prosperity	In other Words
Value of Stock	$P_{S,D}$	$P_{S,P}$	will not be same
Value of Option	0	- ($P_{S,P}$ – C)	focus on payoff to buyer
Total Payoff	$P_{S,D}$	$P_{S,D}$ - ($P_{S,P}$ – C)	s/b same value

- gamma (δ) – number of call options needed t/b written to fully hedge a long stock position
  - fully hedged – goal is to guarantee a payment equal in both states of D & P
  - e.g.,

	Depression	Prosperity	In other Words
Value of Stock	$P_{S,0}$	$P_{S,0}$	will not be same
Value of Option	0	- δ ($P_{S,P}$ – C)	focus on payoff to buyer
Total Payoff	$P_{S,D}$	$P_{S,D}$ – δ($P_{S,P}$ – C)	s/b same value

  - solve for gamma – choose δ such that $P_{S,D}$ = $P_{S,D}$ - δ ($P_{S,P}$ – C)
    - **δ = [$P_{S,D}$ - $P_{S,P}$ / ($P_{S,P}$ – C)]** ~ ratio of range of P_S to range of option payoffs
  - solve for option P
    - ~ P_{Call} = 1/δ [$P_{S,0}$ – [$P_{S,D}$ / (1+r_F)] (~ where PS,D = risk free payout)

  - European Call (1 period binomial)
    - priced as –
      - **a)** fully hedged long position in stock +
      - **b)** writing δ call options w/ strike P of C
    - valuation =
      - **i)** P_{Call} = 1/δ [$P_{S,0}$ – [$P_{S,D}$ / (1+r_F)]
      - **ii)** 1/δ ($P_{S,0}$) - ($P_{S,D}$ / δ C) [C/ (1+r_F)]
      - **iii)** binomial distribution = HR_1 ($P_{S,0}$) – HR_2 [C / (1+r_F)]
        - > where HR_1 = 1 / δ and HR_2 = ($P_{S,D}$ / δ C)

- Black Scholes Option Pricing Model
  - **a)** generally –
    - assumptions – continuous trading in stocks & options + continuously compounded constant r_F + σ of stock return is constant + no dividends + continuous RoR of stocks are normally distributed (~ stock P = log normally distrib) + P moves as random walk
    - ignores – Dividends + American Options
    - European Call -
      - $P_{Call} = N(d_1)\, P_{S,0} - N(d_2)\, (P_{S,0}\, e^{-rFt})C$
      - $P_{Call} = HR_1\, P_{S,0} - HR_2(P_{S,0}\, e^{-rFt})C$ ~ similar to binomial distribution
      - where
        - > $d_1 = [\ln (P_{S,0} / C) + (r_F + (\sigma^2/2))\, t] / \sigma(t^{0.5})$ (note – assumes constant σ, but may not be)
        - > $d_2 = d_1 - \sigma(t^{0.5})$

- implied volatility – compensates for fact that σ <> observable → found w/ iterative approach (no closed form inversion of problem) by - either
  - **a)** estimate from time series data of stock returns
  - **b)** estimate volatility by option premiums observed in market
  - **c)** pure plays

- variables in Option Pricing

Traditional Option	Observable?	Capital Project	
i) C - Strike Price	Y	Amt expended (X)	
ii) σ^2 - Risk of underlying asset	N	σ^2 of project returns	
iii) r_F	Y	r_F	
iv) $P_{S,0}$ – Current P of underlying asset	Y	Value of asset built (S)	
v) t – time to maturity	Y	time can wait w/o losing opp	

- **b)** Dividends -
  - American Call (no div) → no incentive to exercise before maturity (i.e., same as European)
  - American Put (w or w/o div)→ may be incentive to exercise when put is deep in money
  - American Call (small Div) ~ not enough to early exercise → adjust BS to reflect that not entitled to div
    - $P^*_{S,0} = P^*_{S,0}$ – Sum $e^{-rFt}D_t$
  - American Call (large Div) → may be exercised early → see special valuation models if div are known
  - Uncertain Div → not possible to form risk free hedge portfolios → Call options cannot be valued w/o considering risk preferences
- **c)** Warrants –
  - note – dilution effect, but firm gets cash
  - calculation -
    - N = O/S shares
    - M = O/S European warrants (one sh per warrant (t) yrs later at price (C))
    - $W = P_{warrant} = P_{Call} * N / (M+N)$ – accounts for dilution
    - $P_{S,0}* = P_{S,0} + (M/N)W_I$

    - **i)** identify N, M, t, σ, r_F, C, $P_{S,0}$ (known or estimated)
    - **ii)** guess at W_I [e.g., $W_I = N / (M+N) * P_{Call}$ (~ of equivalent warrant)]
    - **iii)** calc $N(d_1)$ & $N(d_2)$
    - **v)** $P_{call} = P_{S,0}* N(d_1) - N(d_2) Ce^{-rFT}$
    - **vi)** $W_E = N / (N+M) * P_{Call}$
    - **vii)** go to step (ii) until $W_B = W_E$

- Real Options
  - Background
    - consider –
      - downsizing (a) is investing against future losses + (b) closes off future opps (forfeits option to resume π ops)
      - opportunity to invest in project is like a call option (investing kills the option's value)
      - in practice → people discount at rates > OCoC
      - substantial value of most cos = options to invest / grow
      - investment = irreversible when
        - > specific to industry (everyone sees as worthless) or company (~sunk costs)
        - > gov't regulation
        - > differences in corp culture
      - option value = EV w/ option – EV w/o option
      - generally better to wait for uncertainty to be resolved
      - acts that create options → more valuable than NPV indicates
      - scale v. flexibility – latter may be nmore valuable in face of uncertainty
      - where large sunk costs exists → closing = opportunity cost b/c lose intangibles
    - cons of NPV =
      - Assumptions → investment is irreversible + now or never proposition + ignores value in creating options through current investment + in practice, WACC is simply used as disc rate

        - leads to missing valuable ops + missing the timing
    - key question = when to exercise option

- option theory of investing – as concept progresses toward implementation → options disappear and value falls(hurdle rate rises)

- Capital Projects
    - valuation methods -
        - **a)** DCF – applies if project cannot be delayed ($t = 0$) or has no variance ($\sigma = 0$)
            - NPV_q = PV (CF) / PV (Capex) → RoT ~ invest if greater than 1
        - **b)** option – when project c/b delayed
            - NPVq – still matters, but is influenced by riskiness of project
            - **option value** = look up % X underlying Asset Value (S) for NPVq and cumulative variance ($\sigma (t^{0.5})$)

    - cumulative variance = $\sigma^2 \times t$ (~ variance of project times time remaining on project)
        - measures how much things could change before time runs out

    - RoT - capital project s/b pursued if PV(CF) > PV (Capex) at maturity of option

IF	NPV	NPVq	$(\sigma^2 t)^{0.5}$	then
i)	< 0	<1	zero	never exercise
ii)	< 0	<1	low	doubtful prospects
iii)	< 0	<1	med / high	less promising – require active development
iv)	< 0	>1	high	very promising b/c high variance
v)	> 0	>1	low / medium	wait if possible
vi)	> 0	> 1	zero	exercise now – will not change

    - consider
        - we can value European Call w/ only cumulative variance and NPV_q
        - option w/ σ or $t = 0$ have no variance and c/b valued w/ DCF
        - simplify complex projects ~ sequence of serially dependent projects
            - **i)** identify main uncertainty
            - **ii)** find upper / lower bounds
        - estimating σ
            - **i)** guess – systematic B and total risk are positively correlated in large samples of operating assets
            - **ii)** estimate – historical data / implied volatility from quoted option P (note – equity returns are levered and more volatile than underlying asset returns)
            - **iii)** simulate σ - Monte Carlo
            - **iv)** locate project w/i "tomato garden" plot – if know whether cumulative variance is high / low

- remember
    - lack of symmetry b/c can't go to negative infinity
    - $P_0 \sim C (1 / 1+r_F)\ t \sim Ce^{-r_F t}$ (if continuous compounding)
    - E(r) return on fully hedged position = r_F → $[P_{S,D} / (P_{S,0} - \delta P_{Call})] - 1 = r_F$
    - Loan Guarantee = Risky Bond + Put on loan
        - $P_{Put} = F(V_0, r_F, T, D, \sigma_V)$
            - where – σ_V^2 = Var (D + E) = Var(D) + Var(E) + 2Cov (D+E)
    - must adjust NPV for opportunity cost of option
    - exercising options kills option value

IX. Multiples Valuation

- Process
  - i) choose comparable firms
    - note – stringent criteria may = too few firms
    - exclude abnormal firms
    - criteria ~ types of goods produced / technology implemented / clientele / size (units produced) / leverage
  - ii) choose bases for multiples → consider industry relevant criteria
    - if can't adjust for capitalization differences → value whole firm v. Operating Income
  - iii) average across industry
  - iv) project bases for valued firm

- Multiples
  - P/E = P/E * EPS
    - trailing earnings – must be discounted one period when applied to future earnings (t+1 is when future earnings become trailing earnings)
    - leading earnings – value future value of firm
    - P_0/EPS_0 = [Div Payout ratio * (1+g)] / (r_E – g)
    - g = retention ratio * ROE = (1-Div payout ratio) * ROE

  - Terminal Values –
    - con ~ assumes past holds for future

  - P / Sales
    - con – may assume too much similarity (lose differentiation further down in F/S)

  - F/A multiples
    - key = how is relative size measured in industry? (~ gross F/A or net F/A)
  -

- remember
  - applying historic multiple to next year ignores future prospects
  - neg earnings → apply multiple to first positive yr and discount back to today
    - can't exclude negative earnings b/c introduces downward bias
    - solution ~ Sum Industry Values / Sum Industry earnings
  - cons – earnings manipulated by accts

X. Portfolio Risk

- generally –
  - is NOT wghted avg of σ of stocks in portfolio (unless stocks are perfectly correlated)
  - calculated by
    - **i)** identify % in each portfolio (x_1 & x_2)
    - **ii)** find σ of individual stocks
    - **iii)** find covariance of individual stocks
      - covariance = either
        - **a)** correlation coefficient$_{12}$ X σ_1 σ_2 X x_1x_2
          - correlation coefficient ~ ???
        - **b)** σ_{12} = expected value of (actual return$_1$ – exp return$_1$) X (act return$_2$ – exp return$_2$)
    - **iv)** weight covariance by proportion of holdings (x_1 & x_2)
    - **v)** multiply squared proportion of holding X variance (for x_1 & x_2 separately)
    - **vi)** portfolio variance = $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(\text{correlation coefficient}_{12}\, \sigma_1\, \sigma_2\, x_1x_2)$

- matrix -

	Stock 1	**Stock 2**	**Stock 3**
Stock 1	$x^2_1\sigma^2_1$	$x_1x_2\sigma_{12}$ = $x_1x_2\, \rho_{12}\, \sigma_1\sigma_2$	$x_1x_3\sigma_{13}$ = $x_1x_3\, \rho_{13}\, \sigma_1\sigma_3$
Stock 2	$x_2x_1\sigma_{21}$ = $x_2x_1\, \rho_{21}\, \sigma_2\sigma_1$	$x^2_2\sigma^2_2$	$x_2x_3\sigma_{23}$ = $x_2x_3\, \rho_{23}\, \sigma_2\sigma_3$
Stock 3	$x_3x_1\sigma_{31}$ = $x_3x_1\, \rho_{31}\, \sigma_3\sigma_1$	$x_3x_2\sigma_{32}$ = $x_3x_2\, \rho_{32}\, \sigma_3\sigma_2$	$x^2_3\sigma^2_3$

- Portfolio Variance = sum of all boxes
- (yellow box) = variance terms
- (white box) = covariance terms