PRICING IN TWO MARKETS/PRICE DISCRIM:

1. Find TR, MR in individual markets

$Q_u = 60 - P_u$	$Q_e = 44 - P_e$
$P_u = 60 - Q_u$	$P_e = 44 - Q_e$
$TRu = 60\,Q_u - Q_u^2$	$TRe = 44\,Q_e - Q_e^2$
$MR_u = 60 - 2Q_u$	$MR_e = 44 - 2Q_e$

2. $MR_u = MR_e$
$60 - 2Q_u = 44 - 2Q_e$
$Q_u = 8 + Qe$
$Q_e = Q_u - 8$

3. ATC then find TC then find MC_T in markets combined
$ATC = 4 + 0.1Q_T$
$TC = 4Q_T + 0.1Q_T^2$
$MC_T = 4 + 0.2Q_T$

Sub in Qe and Qu for Q_T
$MC_T = 4 + 0.2Q_T = 4 + .2Qu + .2Qe$

Sub Qu variable
Get MC_T in terms of 1 variable
$MC_T = 4 + .2(8 + Qe) + .2Qe = 5.6 + .4Qe$
(Note: you only need to do this step when there are variables in the MC equation; if MC=30000 or some constant, just MRs=MC and solve.)

4. MC_T = MR of 1 market
$MC_T = MR_e$
$5.6 + .4Qe = 44 - 2Q_e$
Solve $Q_e = 16$
Sub into Qu equation $Q_u = 8 + Qe = 24$

5. Find P's
Qu=24, so Pu=60-Qu = 36
Qe=16, so $P_e = 44 - Q_e = 28$

QUASI-COMPETITIVE SOLUTION

Key words: price war
$P = 100 - .5(Q_1 + Q_2)$
$TC_1 = 5Q_1$ and $TC_2 = .5Q_2^2$
$MC_1 = 5$ $MC_2 = Q_2$

Set P = MC_1 and solve for Q_1
$100 - .5(Q_1 + Q_2) = 5$
$Q_1 = 190 - Q_2$

Set P = MC_2, sub in Q_1 and solve for Q_2
$100 - .5[(190 - Q_2) + Q_2] = Q_2$
$Q_2 = 5$

Solve for Q_1 and then P
$Q_1 = 190 - Q_2$ or $Q_1 = 185$
$P = 100 - .5(Q_1 + Q_2) = 100 - .5(185 + 5)$
$P = 5$

Calculate Profits
$\pi_1 = TR_1 - TC_1 = 85(5) - 5(185) = 0$
$\pi_2 = TR_2 - TC_2 = 5(5) - .5(5^2) = 12.5$

2 PART TARIFFS:

Two Parts: Use Fee (MC) and Entry Fee (CS)

Demand curve for Concerts P=150-3Q
TC produce Orchestra concert $TC = 100 + 30Q + 0.5Q^2$
Currently price charged = $60

Calculate Q for existing price
P=150-3Q, P=60, so Q = 30

Calculate current TR and TC:
TR = PQ = 30*60 = $1,800
$TC = 100 + 30(30) + 0.5(30)^2 = 1{,}450$

New situation
Find MC: MC = 30+Q = 60
Set P = MC = <u>Use Fee</u>
150 - 3Q = 30+Q
Q = 30
Solve for P: P = 150 - 3Q = 60
Go to triangle and plug
CS = .5bh = numbers from triangle
= .5(30)(90) = 1,350

new price structure:
use price = 60
entry fee = CS = 1,350

new profit = TR – TC
=PQ + entry fee – TC function plug Q in
= (60)(30) + 1,350 – 1,450 = 1,700

old profit = TR - TC
=PQ – TC function plug Q in
= (60)(30) – 1,450 = 350

PROFIT DIFFERENCE = NEW – OLD = 1,700 - 350

COURNOT SOLUTION

Key words: quantity comparison
$P = 100 - .5(Q_1 + Q_2)$
$TC_1 = 5Q_1$ and $TC_2 = .5Q_2^2$
$MC_1 = 5$ $MC_2 = Q_2$

i) *Firm 1:*
a) $TR_1 = PQ_1 = [100 - .5(Q_1 + Q_2)]\,Q_1$
$TR_1 = 100Q_1 - .5Q_1^2 - .5Q_1Q_2$
$MR_1 = 100 - Q_1 - .5Q_2$

b) Set $MC_1 = MR_1$ or $100 - Q_1 - .5Q_2 = 5$
$Q_1 = 95 - .5Q_2$

ii) *Firm 2:*
i) $TR_2 = PQ_2 = [100 - .5(Q_1 + Q_2)]\,Q_2$
$TR_2 = 100Q_2 - .5Q_2^2 - .5Q_1Q_2$
$MR_2 = 100 - Q_2 - .5Q_1$

ii) Set $MC_2 = MR_2$ or $100 - Q_2 - .5Q_1 = Q_2$
$Q_2 = 50 - .25Q_1$

Sub Q_1 into Q_2 and solve for Q_1 and Q_2, then price:

$Q_2 = 50 - .25Q_1$	$= 50 - .25(95 - .5Q_2)$	$= 30$
$Q_2 = 30$		
$Q_1 = 95 - .5Q_2$	$= 95 - .5(30)$	$= 80$
$P = 100 - .5(Q_1 + Q_2)$	$= 100 - .5(80 + 30)$	= $45

Calculate profits:

$\pi_1 = TR_1 - TC_1$	$= 80(45) - 5(80)$	= $3200
$\pi_2 = TR_2 - TC_2$	$= 30(45) - .5(30^2)$	= $900

STACKELBERG

Key words: leader, follower (just 2 firms)
$P = 100 - .5(Q_1 + Q_2)$
$TC_1 = 5Q_1$
$MC_1 = 5$

<u>Firm 1</u>
$TR = PQ_1 = 100Q_1 - .5(Q_1 + Q_2)Q_1$
$\pi_1 = TR_1 - TC_1$
$\pi_1 = 100 - .5Q_1^2 - .5Q_1Q_2 - 5Q_1$

Sub in Cournot reaction functions
$Q_2 = 50 - .25Q_1$
$\pi_1 = 70\,Q_1 - .375Q_1^2$
Take deriv of π_1 $0 = 70 - .75\,Q_1$
Solve for $Q_1 = 93.33$
Sub in Q_1 and solve for π_1

<u>Firm 2</u>
Sub Q_1 into Q_2 reaction function
Solve for Q_2

COLLUSIVE SOLUTION

Key words: collusion or cooperate
$P = 100 - .5(Q_1 + Q_2)$
$TR = [100 - .5(Q_1 + Q_2)]\,(Q_1 + Q_2)$

$TC_1 = 5Q_1$ and $TC_2 = .5Q_2^2$
$MC_1 = 5$ $MC_2 = Q_2$

Calculate profit:
$\pi = TR - TC$
$\pi = TR - (5Q_1 + .5Q_2^2)$
$\pi = 95Q_1 + 100\,Q_2 - .5Q_1^2 - Q_1Q_2 - Q_2^2$

Take partials and set equal to zero:
$0 = 95 - Q_1 - Q_2$ wrt Q1
$0 = 100 - Q_1 - 2Q_2$ wrt Q2
Solve for Q_1:
$Q_1 = 95 - Q_2$

Sub in Q_1, solve for Q_2, Q_1 and price:

$Q_2 = 50 - .5(Q_1)$	$= 50 - .5(95 - Q_2)$	$= 5$
$Q_1 = 95 - Q_2$	$= 95 - 5$	$= 90$
$P = 100 - .5(Q_1 + Q_2)$	= $52.5	

Calculate profits:

$\pi_1 = TR_1 - TC_1$	$= 52.5(90) - 5(90)$	$= 4275$
$\pi_2 = TR_2 - TC_2$	$= 52.5(5) - .5(5^2)$	$= 250$

PRICE LEADER:

Key words: fringe, many small firms, cartel
$Q_{Total} = Q_{Leader} + Q_{Fringe}$
$Q_{Leader} = Q_{Total} - Q_{Fringe}$
$P_F = MR_F = MC_F$ for small firms
$MR_{ALL} = MC_{ALL}$ if profit maximizing

$Q_T = 25{,}000 - 30P$
$TC_L = 90{,}000 + 200Q_L + .25Q_L^2$
$MC_F = 90 + 0.04Q_F$

Solve for Q_F (if not given in problem):
$P_F = MR_F = MC_F$
$P_F = 90 + 0.04Q_F$
Rewrite $Q_F = 25P - 2{,}250$

Solve for Q_L:
$Q_L = Q_T - Q_F$
$Q_L = (25{,}000 - 30P) - (25P - 2{,}250)$
Rewrite $P_L = 495.455 - .01818Q_L$

Find TR_L and MR_L:
$TR_L = P_LQ_L$
$TR_L = (495.455 - .01818Q_L)(Q_L)$
$MR_L = 495.455 - .03636Q_L$

$MR_L = MC_L$:
MC_L = deriv of TC_L
$495.455 - .03636Q_L = 200 + .05Q_L$
$Q_L = 3421.2019$

Substitute in for P_L:
$P_L = 495.455 - .01818Q_L = 433.26$

TRANSFER PRICING

A firm has 2 divisions; one needs products from other

No External Market
Find profit maximizing transfer price as follows:
- Find MR_F from demand D_F
- Find firm's $MC_F = MC_1 + MC_2$
- Set $MR_F = MC_F$ and solve for P_F and Q_F
- Sub Q_F for Q_1 in the MC_1 function, solve for MC_1

$P_F = 1{,}000 + .1Q_F$
$MC_1 = 10 + .01Q_1$ $MC_2 = 100 + .05Q_2$
(where $Q_F = Q_1 = Q_2$ due to a 1-for-1 demand)

$TR_F = 1{,}000\,Q_F + .1Q_F^2$
$MR_F = 1{,}000 + .2Q_F$

$MC_F = MC_1 + MC_2 = (10 + .01Q_1) + (100 + .05Q_2)$
$= (10 + .01Q_F) + (100 + .05Q_F)$
$= 110 + .06Q_F$

$MR_F = MC_F$ where $1{,}000 + .2Q_F = 110 + .06Q_F$
$Q_F = 3423$

$MC_1 = 10 + .01Q_1 = 10 + .01Q_F$
$MC_1 = \$44.23$ = transfer price

Perfectly Competitive External Market
Find profit maximizing transfer price as follows:
- Set $MR_1 = MC_1$ *(to determine best output for div. 1, Q_1)*
- Estimate div. 2 demand and determine MR_2
- Find div. 2's marginal cost: MC_F = xfer price + MC_2
- Find div. 2 max output: $MC_F = MR_2$ *(if Q2<Q1, then div 1 will sell excess in external market ?)*

Example:
Div 1 makes shirts, Div 2 markets them. Marketing costs = $3/shirt in the competitive market. Find optimal output for Div 1 and results wrt the market.

$P_1 = 12$
$TC_1 = 3{,}000 + 2Q_1 + .001Q_1^2$

$P_2 = 40 - .01Q_2$
$TC_2 = 1{,}700 + 15Q_2$ *(fixed cost = 1700, variable cost of 3 for mktg and 12 for transferring per shirt)*

$TR_1 = P_1Q_1 = 12Q_1$
$MR_1 = 12$
$MC_1 = 2 + .002\,Q_1$

$MR_1 = MC_1$ or $12 = 2 + .002Q_1$
$Q_1 = 5{,}000$

$\pi = TR_1 - TC_1 = 12Q_1 - (3{,}000 + 2Q_1 + .001Q_1^2)$
$\pi = 22{,}000$

$TR_2 = P_2Q_2 \quad = 40Q_2 - .01Q_2^2$
$MR_2 = 40 - .02Q_2$
$MC_2 = 15$

$MR_2 = MC_2$ or $40 - .02Q_2 = 15$
$Q_2 = 1250$

$Q_1 < Q_2$, thus Div 1 will sell 1250 units to Div 2 and the rest on the open market

MORAL HAZARD W/ COMPENSATION:

$EU = p_1U_1 + p_2U_2$
Utility function must be given
Worker utility = W + B – C + wages + bonus - cost

Solve for utility when work hard:
W + B – C

Solve for utility when shirk:
W + B – C

Compare utility to determine worker behavior:
Choose greatest utility

Determine firm π levels under hard and shirk
Sum of probabilities of profits
May be some other factor

Determine effect of compensation on firm π:
Firm gets π that matches what worker does
Subtract wages from π to determine real π

Determine optimal compensation:
EU work hard = just above EU shirk
$EU_{HARD} = EU_{SHIRK}$
adjust so $EU_{HARD} = EU_{SHIRK}^+$

WARRANTY:

Sally's product = Excellent and Joe's product = Poor
P_s = $160, if seen as high quality
P_j = $120,if seen as poor quality
C_s= $80 and C_j=$80

Info. is imperfect, and warranty signals high quality.
Warranty C_s= $30 and Warranty C_j=$80

Sally's Profit w/out warranty= $160-$80 = $80
Joe's Profit w/out warranty = $120-$30 = $90

Sally will issue a warranty if
$160 - $80 - $5Y > $80 (profit w/out warranty)
8 > Y thus, Sally can issue warranty of up to 8 yrs.

Joe will issue a warranty if
$160 - $30 - $10Y > $90 (profit w/out warranty)
4 > Y thus, Joe can issue a warranty of up to 4 yrs.

If Sally issues a warranty of 4+ yrs, she'll make
$160 - $80 - $5*$4^+$ = $$60^-$ per unit

If they collude for maximum profits:
Sally issues a warranty for 1 day signaling high quality, making her profits:
$160 - $60 – $5 (1/365) = $$80^-$

Joe does not issue a warranty, making same profit of
$120 - $30 = $90

INSURANCE:

House is worth 350 (property) and 300 (financial assets). There is 1% chance of a fire causing loss.

Utility = $U = W^{0.5}$

Exp. Utility = $0.99\,(650)^{0.5} + 0.01\,(300)^{0.5} = 25.413$

What's the maximum willing to spend on insurance?
Set Utility with Insurance 650-P = Existing Utility
$U = (650 – P)^{0.5} = 25.413$
Square both $(650 – P) = (25.413)^2 \Rightarrow P = 4.162$

*Note this is above the expected loss (350*0.1 = 3.5) You can expect to pay more than

price to MR = same intercept 2X slope
MR to price = same intercept 1/2 slope

POSTED PRICE VS. DUTCH AUCTION:

You wish to maximize rev. by buying back stock. Market response probabilities follow: good (0.35); neutral (0.40); and bad (0.25). Ace values stock at 40.

Set up table with

			0.35	0.40	0.25
Val. Ace	Price	Profit/ Share	Profit Good	Profit Neut.	Profit Bad
40	35	5	10 (50)	7 (35)	5 (25)
40	36	4	12 (48)	10 (40)	8 (32)
40	37	3	14 (42)	12 (36)	11(33)
40	38	2	16 (32)	13 (26)	12 (24)

(*=No. in () = Profit/Share*Number of Shares

By announcing price before shares are tendered, the expected profit is

Price	Expected Profit
35	0.35*50 + 0.40*35 + 0.25*25 = 37.75
36	0.35*48 + 0.40*40 + 0.25*32 = 40.80
37	0.35*42 + 0.40*36 + 0.25*33 = 37.35
38	0.35*32 + 0.40*26 + 0.25*24 = 27.60

In a modified Dutch auction, price is announced after shares are tendered. Ace will buy back at 35 if conditions are good, 36, if conditions are neutral, and at 37 if conditions are bad.

0.35*50 + 0.40*40 + 0.25*33 = 41.75

41.75 > 40.80 = 0.95 (Value of Information)

If risk averse and $U = \pi^{.4}$, announce price before or after?
Do expected utilities at each price level, ex:
35 $\quad 0.35*50^{.4} + 0.40*35^{.4} + 0.25*25^{.4} = 4.238$
do for 36, 37, 38
pick winner

Do expected utility of Dutch:
$0.35*50^{.4} + 0.40*40^{.4} + 0.25*33^{.4} = 4.435$

compare winner to Dutch, choose higher utility

2 PART TARIFFS:

Two Parts: Use Fee (MC) and Entry Fee (CS)

Demand curve for Concerts P=150-3Q
TC produce Orchestra concert $TC=100+30Q+0.5Q^2$
Currently price charged = $60

Calculate Q for existing price
P=150-3Q, P=60, so Q = 30

Calculate current TR and TC:
TR = PQ = 30*60 = $1,800
$TC = 100 + 30\,(30) + 0.5\,(30)^2 = 1,450$

New situation
Find MC: MC = 30+Q = 60
Set P = MC = <u>Use Fee</u>
150 - 3Q = 30+Q
Q = 30
Solve for P: P = 150 - 3Q = 60
Go to triangle and plug
CS = .5bh = numbers from triangle
= .5(30)(90) = 1,350

new price structure:
use price = 60
entry fee = CS = 1,350

new profit = TR – TC
=PQ + entry fee – TC function plug Q in
= (60)(30) + 1,350 – 1,450 = 1,700

old profit = TR - TC
=PQ – TC function plug Q in
= (60)(30) – 1,450 = 350

profit difference = new – old = 1,700 - 350

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$Q_F = 3423$

$MC_1 = 10 + .01Q_1 \quad = 10 + .01Q_F$
MC_1 = $44.23 = transfer price

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Find profit maximizing transfer price as follows:
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$MR_1 = MC_1$ or $12 = 2 + .002Q_1$
$Q_1 = 5,000$

$\pi = TR_1 – TC_1 \quad = 12Q_1 – (3,000 + 2Q_1 + .001Q_1^2)$
$\pi = 22,000$

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